1 Profit Max

$f(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$

What is the cheapest way to produce output y?

$w_1 = 1, w_2 = 1$

$x_1^{\frac{1}{2}} x_1^{\frac{1}{2}} = y, x_1 = y, x_2 = y$

$$y + y = 2y$$

$$c(y) = 2y$$

1.1 Price Taking

$$\pi(y) = y[p(y)] - c(y)$$

If a firm is a very small player in a very larger market, it is reasonable for them to assume any output they produce will have a negligible impact on the price of that good. **"Perfect Competition"**

Price-taking assumption. $p(y) = p$

The price they get for output is fixed and does not depend on their choice of output.

$$\pi(y) = yp - c(y)$$

1.2 Maximizing Profit

$$\pi(y) = yp - c(y)$$

To maximize this, find where it's derivative is zero. We need to find the output y^* whre the marginal profit is zero.

$$\frac{\partial(\pi(y))}{\partial y} = \frac{\partial(yp - c(y))}{\partial y} = 0$$

$$\frac{\partial(yp)}{\partial y} - \frac{\partial(c(y))}{\partial y} = 0$$

$$\frac{\partial (yp)}{\partial y} = \frac{\partial (c(y))}{\partial y}$$

$$MR = MC$$

This is the profit maximizing condition for **any firm** in any kind of market.

Suppose this was not ture:

$$MR > MC$$

Increasing output will increase profit since revenue will go up more than cost.

$$MC > MR$$

I can increase profit by lowering output and save more (MC) than I lose in revenue.

So any firm, we have the optimization conditoin:

$$MR = MC$$

Under price taking:

$$p = MC$$

1.3 Example 1

Output price is $p = 10$ and cost is $c(y) = y^2$. What is the optimal output?

$$\pi(y) = 10y - y^2$$

$$\pi'(y) = \frac{\partial (\pi(y))}{\partial y} = 10 - 2y$$

$$10 - 2y = 0$$

$$10 = 2y$$

$$y^* = 5$$

$$\pi(5) = 10 * 5 - 5^2 = 50 - 25 = 25$$

1.4 Example 2

Output price is 10 and cost is $c(y) = y^2 + 5y$. What is the optimal output?

$$\pi(y) = 10y - y^2 - 5y$$

$$5y - y^2$$

$$\frac{\partial\left(5y - y^2\right)}{\partial y} = 5 - 2y$$

$$5 - 2y = 0$$

$$2y = 5$$

$$y = 2.5$$

1.5 What can go wrong?

No Maximum:

Output price is 10 and cost is $c(y) = 2y$

$$\pi(y) = 10y - 2y = 8y$$

$$8 = 0$$

Output price is 1 and cost is is $c(y) = 2y$

$$\pi(y) = y - 2y = -y$$

"Optimal" is a Minimum:

Output price is 10 and cost is $c(y) = 100y^{\frac{1}{2}}$

$$\pi(y) = 10y - 100y^{\frac{1}{2}}$$

$$\frac{\partial(\pi(y))}{\partial y} = 10 - 50y^{-\frac{1}{2}}$$

$$10 - 50\frac{1}{\sqrt{y}} = 0$$

$$10 = 50\frac{1}{\sqrt{y}}$$

$$\sqrt{y} = 5$$

$$y = 25$$

Because the cost function does not have increasing marginal cost, this is actually the **cost minimizing** level output, not the cost maximizing level of output.

2 Monopoly

2.1 Relaxing the Price-Taking Assumption

A monopolist is a firm that is the only seller of some product.

Since they are the only seller, if they want to sell more, they need to lower the price.

What is the relationship between price and output for a monopolist?

$$p(y)?$$

The inverse market demand function tells us the most that consumer would be willing to pay for a certain number of units.

$$y = 100 - p$$

$$p = 100 - y$$

If they monopolist wants to sell 50 units, the most they can charge is 50.

For a monopolist, the function $p(y)$ is simply the market inverse demand.

2.2 Monopolist Profit Function

For a monopolist the profit function is:

Where $p(y)$ is the market inverse demand:

$$\pi(y) = yp(y) - c(y)$$

2.3 Example

Demand: $100 - p$. $c(y) = 10y$

a) What is the inverse demand?

$$y = 100 - p$$

Inverse demand:

$$p = 100 - y$$

b) What is the optimal y?

$$\pi(y) = y(100-y) - 10y$$

$$\pi(y) = 100y - y^2 - 10y = 90y - y^2$$

$$\frac{\partial(90y - y^2)}{\partial y} = 0$$

$$90 = 2y$$

$$y^* = 45$$

c) What price does the monopolist charge?

Evaluate the inverse demand at y^*:

$$100 - 45 = 55$$

$$p^* = 55$$

d) What is their profit?

$$45 * 55 - 10(45) \quad = \quad 2025$$

2.4 You Try

Demand: $100 - p$. $c(y) = y^2$

a) What is the inverse demand?

$$p = 100 - y$$

b) What is the optimal y?

$$\pi(y) = y(100-y) - y^2$$

$$100y - y^2 - y^2$$

$$100y - 2y^2$$

$$\frac{\partial\left(100y - 2y^2\right)}{\partial y} = 0$$

$$100 - 4y = 0$$

$$4y = 100$$

$$y = 25$$

$$\begin{pmatrix} 5 & 450 \\ 10 & 800 \\ 15 & 1050 \\ 20 & 1200 \\ 25 & 1250 \\ 30 & 1200 \\ 35 & 1050 \\ 40 & 800 \\ 45 & 450 \\ 50 & 0 \\ 55 & -550 \end{pmatrix}$$

c) What price does the monopolist charge?

$$100 - y$$

$$100 - 25 = 75$$

$$p^* = 75$$

d) What is their profit?

$$75 * 25 - 25^2 \quad = \quad 1250$$

2.5 Monopolist and Elasticity

The optimal level of output for a monopolist never occurs where demand is inelastic.

Monopolists love inelastic demand.

What happens if they increase price by 1%?

Price goes up by 1% and demand goes down by less than 1% what happens to revenue?

The revenue goes up!

Costs go down because the firm sells less!

Profit has to go up!

At the optimum y^* demand will be elastic (if it can be).

2.6 Markup

2.6.1 The Markup Equation

2.6.2 Checking for Our Example

2.6.3 In Practice

Example 1: Suppose $\epsilon = -2$ and $mc = 10$. What does the monopolist charge?

Example 2: Suppose $\epsilon = -1.5$ and $p = 100$. What is the monopolist's marginal cost?

2.7 Surplus and Deadweight Loss

Demand: $100 - p$. $c(y) = 10y$