1 Short Run Cost

$f(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$

Let $w_1 = 1, w_2 = 1$.

Conditional Factor Demands: $x_1^* = y, x_2^* = y$.

$$f(y, y) = \sqrt{y}\sqrt{y} = y$$

How much does it cost to use the optimal amount of x_1, x_2 to produce y?

$$c(y) = y + y = 2y$$

However, suppose x_2 number of machines in a factor. $x_2 = 4$ in the **short run.**

The short function production function is:

$$f(x_1 4) = x_1^{\frac{1}{2}} 4^{\frac{1}{2}} = 2x_1^{\frac{1}{2}}$$

There is only one amount of x_1 that will produce output y when $x_2 = 4$.

How much x_1 do you need to produce y?

$$2x_1^{\frac{1}{2}} = y$$

$$x_1^{\frac{1}{2}} = \frac{1}{2}y$$

$$x_1 = \frac{1}{4}y^2$$

$$f\left(\frac{1}{4}y^2, 4\right) = \sqrt{\frac{1}{4}y^2}\sqrt{4} = 2\frac{1}{2}y = y$$

The cost of using $x_1 = \frac{1}{4}y^2$ and $x_2 = 4$ is

$$w_1 \frac{1}{4}y^2 + w_2 4$$

Since $w_1 = 1, w_2 = 1$

$$c(y) = \frac{1}{4}y^2 + 4$$

Let's compare the short run and long run cost:

1	2.	4.25
2	4.	5.
3	6.	6.25
4	8.	8.
5	10.	10.25
6	12.	13.
7	14.	16.25
8	16.	20.
9	18.	24.25
10	20.	29.

This shows that the short-run cost with $x_2 = 4$ is higher everywhere except for $y = 4$ where $x_2 = 4$ is the optimal amount of x_2 anyway.

2 Profit Max

With cost minimization, the firm summarizes everything they need to know about prouction into the cost function $c(y)$ which tells us the lowest cost possible for producing output y.

With the cost function, we can produce a profit function that is a function of **only** y.

$$\pi(y) = revenue(y) - c(y)$$

$$\pi(y) = yp(y) - c(y)$$

There are many different assumption you can make about firms, the demand, and the market environment that they are in that lead to different $p(y)$ functions.

2.1 Price Taking

The simplest assumption is that p does not depend on y.

If a firm is a very small player in a very large market, any amount of output they produce will have negligibe impact on the market for that good. In this case, that reasonably assumption that market price does not depend on their output.

$$p(y) = p$$

We call this the **price-taking assumption.** We say that markets where this assumption is reasonable are in **perfect competition**.

$$\pi(y) = py - c(y)$$

2.2 Maximizing Profit

The optimal y has to occur where the derivative of $\pi(y)$ with respect to y is zero:

$$\pi'(y) = \frac{\partial(\pi(y))}{\partial y} = 0$$

Suppose marginal profit wasn't zero:

$$\pi'(y) > 0$$

If marginal profit is positive, then increasing y by a little will increase profit by $\pi'(y)$.

$$\pi'(y) < 0$$

You can decrease output and profit will go up.

At the optimum, this to be true:

$$\pi'(y) = 0$$

Since $\pi = rev - cost$

Let MR be the derivative of the revenue part of the profit function and let mc bet $\frac{\partial(c(y))}{\partial y}$. The condition $\pi'(y) = 0$

$$mr - mc = 0$$

$$mr = mc$$

Marginal revenue has to be equal to marginal cost.

Suppose otherwise:

$$mr > mc$$

Increasing output will increase revenue faster than cost and profit will go up.

$$mr < mc$$

Decreasing output will decrease costs more than it will decrease revenue. Save more in cost than you lose in revnue and profit goesup.

This is true **for every firm** regardless of the assumptions we make about the market.

$$mr = mc$$

Under the price-taking assumption / in perfect competition specifically, marginal revnue $mr = p$.

$$p = mc$$

2.3 Example 1

Output price is $p = 10$ and cost is $c(y) = y^2$. What is the optimal output?

Construct the profit function:

$$\pi(y) = 10y - y^2$$

$$\frac{\partial(\pi(y))}{\partial y} = 10 - 2y$$

Set this to zero to find the optimal:

$$10 - 2y = 0$$

$$y = 5$$

Here are the profits possible at $y = 1, 2, 3..., 10$:

$$\{9., 16., 21., 24., 25., 24., 21., 16., 9., 0.\}$$

At $y = 5$ this profit is maximized and the profit is:

$$\pi(5) = 10 * 5 - 5^2 = 50 - 25 = 25$$

2.4 Example 2

Suupose a firm makes the price-taking assumption and output price is $p = 10$ and cost is $c(y) = y^2 + 5y$. What is the optimal output?

a) Set up the profit function.

$$\pi(y) = 10y - \left(y^2 + 5y\right)$$

b) Find the optimal output.

$$\frac{\partial(\pi(y))}{\partial y} = 10 - 2y - 5$$

$$10 - 2y - 5 = 0$$

$$2y = 5$$

$$y^* = \frac{5}{2}$$

2.5 What can go wrong?

No Non-Trivial Maximum

Output price is 10 and cost is $c(y) = 2y$

$$\pi(y) = 10y - 2y = 8y$$

$$\frac{\partial(\pi(y))}{\partial y} = 8$$

$$8 \neq 0$$

Output price is 10 and cost is $c(y) = 12y$

$$\pi(y) = 10y - 12y = -2y$$

The best you can do here is produce 0.

"Optimal" is a Not a Maximum:

Output price is 10 and cost is $c\left(y\right)=100y^{\frac{1}{2}}$

$$\pi\left(y\right)=10y-100y^{\frac{1}{2}}$$

$$10-\frac{1}{2}100y^{-\frac{1}{2}}=0$$

$$y=25$$

$$\{0.,-173.607,-216.228,-237.298,-247.214,-250.,-247.723,-241.608\}$$

Notice that $y=25$ gives -250 profit which is actually the **minimum.** That's because this profit function does not have a concave shape. It is not a mountain, it is a valley.

3 Monopoly

A monopolist is a firm that sells a good for which there are no good substitutes.

They are a firm that can charge whatever they want as long as consumers will pay for it. They are limited by competition, they are only limited by demand.

$$\pi\left(y\right)=p\left(y\right)y-c\left(y\right)$$

Because a monopolist can charge as much as consumers are willing to pay, the $p\left(y\right)$ is simply what consumers are willing to pay for output y.

$p\left(y\right)$ is simply the inverse demand function: the most consumers will pay for y.

3.1 Example

Demand: $100-p$. $c\left(y\right)=10y$

a) What is the inverse demand?

$$y=100-p$$

Inverse demand:

$$p=100-y$$

b) What is their profit function:

$$\pi\left(y\right) = \left(100 - y\right) y - 10y$$

$$100y - y^2 - 10y$$

$$\pi\left(y\right) = 90y - y^2$$

$$\frac{\partial\left(90y - y^2\right)}{\partial y} = 90 - 2y$$

$$90 - 2y = 0$$

$$2y = 90$$

$$y^* = 45$$

c) What price does the monopolist charge?

Evaluate the inverse demand at $y = 45$

$$p^* = 100 - 45 = 55$$

d) What is their profit?

$$55 * 45 - 10 * 45 \quad = \quad 2025$$

3.2 You Try

Demand: $y = 100 - p$ and cost is $c\left(y\right) = y^2$

a) What is the inverse demand?

$$y = 100 - p$$

$$p = 100 - y$$

b) What is the optimal y?

$$\pi(y) = (100 - y)\, y - y^2$$

$$100y - y^2 - y^2$$

$$100y - 2y^2$$

$$\frac{\partial\left(100y - 2y^2\right)}{\partial y} = 100 - 4y$$

$$100 = 4y$$

$$y^* = 25$$

c) What price does the monopolist charge?

$$p = 100 - y$$

$$p^* = 75$$

d) Calculate the profit:

$$\pi(y) = (100 - 25) * 25 - 25^2 \quad = \quad 1250$$

3.3 Monopolist and Elasticity

3.4 Markup

3.4.1 The Markup Equation

3.4.2 Checking for Our Example

3.4.3 In Practice

Example 1: Suppose $\epsilon = -2$ and $mc = 10$. What does the monopolist charge?

Example 2: Suppose $\epsilon = -1.5$ and $p = 100$. What is the monopolist's marginal cost?

3.5 Surplus and Deadweight Loss

Demand: $100 - p$. $c(y) = 10y$