0.1 Two-Part Tariff

Demand for each consumer: $y = 10 - p$.

Marginal cost is 2 so that for each consumer, $c(y) = 2y$.

a) What quantity and price would maximize the profit for each individual consumer?

$$p = 10 - y$$

Consust the profit function the firm earns for each consumer:

$$\pi(y) = y(10 - y) - 2y$$

$$\frac{\partial(y(10 - y) - 2y)}{\partial y} = 8 - 2y$$

$$8 = 2y$$

$$y^* = 4$$

$$p^* = 10 - 4 = 6$$

$$\pi = 4 * 6 - 4 * 2 = 16$$

$$\pi^* = 16$$

Suppose they sold coffe at the lowest price the would willing to maximize the consumer's surplus.

If the firm sells a mug (membership fee) for \$32 (capturing all of the consumer surplus) which gives the consumer the right to buy coffee at \$2 per unit. Their profit is \$32.

Two-part tarrif.

Let's suppose cost was zero in this case.

$$y = 10 - p$$

Inverse demand:

$$p = 10 - y$$

Costs are zero. $c(y) = 0$

The consumer surplus from free coffee is $50. So the firm can charge an up-front fee of $50 for the right to get free coffee. This gives them a profit of $50.

Compare this to the optimal traditional pricing:

$$\pi(y) = y(10 - y)$$

$$y^* = 5, p^* = 5$$

$$\pi(5) = 5 * 5 = 25$$

Earn $25 per consumer by using two-part tarrif.

amusment parks are the best example of this, amc movie pass,

1 Cournot Oligopoly

$$\pi() = qp(q) - c(q)$$

In monopoly model, the monopoly was the only firm that serves the market. There, it is clear how to figure out the price given the chosen quantity: it is what consumers will pay for q.

Now we will look at markets where there is competition, but not enough to justify price-taking, what should we do with the $p(q)$?

Assume three are multiple firms N. The firms are "named" $i \in \{1, 2, 3..., N\}$. Each firm chooses their quantity q_i

The **market quantity** is $Q = q_1 + q_2 + ... + q_N$.

The price they charge p is the most consumers are willing to pay, **for the market quantity.**

The price they firms chrge is $p(Q)$ where $p()$ is the inverse demand function.

A **firms profit function** in cournot oligopoly:

$$\pi(q_i, Q) = q_i p(Q) - c(q_i)$$

1.1 Example with Two Firms

$p(Q) = 100 - Q$, $c(q_i) = 10q_i$

$Q = q_1 + q_2$

Profit function of firm 1:

$$\pi_1(q_1, Q) = q_1(100 - Q) - 10q_1$$

What we want to do is have the firm maximize this:

$$\pi_1(q_1, Q) = q_1(100 - Q) - 10q_1$$

$$= q_1(100 - (q_1 + q_2)) - 10q_1$$

Suppose we know what q_2 was.

Suppose firm 1 knew $q_2 = 50$

$$= q_1(100 - (q_1 + 50)) - 10q_1$$

$$= 100q_1 - q_1^2 - 50q_1 - 10q_1$$

$$= 40q_1 - q_1^2$$

10	300
20	400
30	300
40	0
50	-500

Clerarly the profit is going up and then down. It has a concave shape. We can find the optimal quantity by looking for where marginal profit is zero:

$$\frac{\partial\left(40q_1 - q_1^2\right)}{\partial q_1} = 40 - 2q_1$$

$$40 - 2q_1 = 0$$

$$q_1 = 20$$

If firm 1 knows $q_2 = 50$, the best they can do is choose $q_1 = 20$.

Suppose firm 1 knew $q_2 = 10$

$$= q_1 (100 - (q_1 + 10)) - 10q_1$$

Where is this maximized (what is the optimal q_1 given that $q_2 = 10$)

$$q_1 = 40$$

In summary, if $q_2 = 50$ firm 1 wants to choose $q_1 = 20$

if $q_2 = 10$ firm 1 wants to choose $q_2 = 40$

1.2 Best Responses

These are best responses.

Firm 1's best response to $q_2 = 50$ is $q_1 = 20$

Firm 1's best response to $q_2 = 10$ is $q_1 = 40$

Here, each firms profit function and thus their optimal strategy (choice of quantity) depends on the choice of the other players firms.

1.3 What is a game:

A game consists of three things:

Players,

Strategies,

Payoffs given the strategies of all the players.

1.3.1 Nash Equilibrium

What we don't know what firm 2's strategy is? How should we solve this game?

Nash equilibrium is the most common solution concept in game theory.

To motivate this, let's solve for the **best response function** for each firm.

Best response function for any quantity for the other firm, what quantity should I choose?

$$\pi_1(q_1, q_2) = q_1(100 - (q_1 + q_2)) - 10q_1$$

Let's simplify this profit function:

$$= 100q_1 - q_1^2 - q_1 q_2 - 10q_1$$

$$= 90q_1 - q_1^2 - q_1 q_2$$

Where is this maximized? Where is the derivative of this function with respect to their quantity q_1 equal to zero?

$$\frac{\partial\left(90q_1 - q_1^2 - q_1 q_2\right)}{\partial q_1} = 0$$

$$90 - 2q_1 - q_2 = 0$$

Solve this for q_1 :

$$2q_1 = 90 - q_2$$

$$q_1 = 45 - \frac{1}{2}q_2$$

This is the **best response function** for firm 1.

Let's check it based on the points we looked at earlier:

Firm 1's best response to $q_2 = 50$ is $q_1 = 20$

Firm 1's best response to $q_2 = 10$ is $q_1 = 40$

$$q_1 = 45 - \frac{1}{2}(50) = 20$$

$$q_1 = 45 - \frac{1}{2}(10) = 40$$

Firm 2's best response:

$$\pi_2(q_1, q_2) = q_2(100 - (q_1 + q_2)) - 10q_2$$

Maximizing this, we will find that the optimal q_2 :

$$q_2 = 45 - \frac{1}{2}q_1$$

Let's think about a pair of strategies we have looked at in terms of best response.

Our original thought experiment involved firm one thinking firm two would choose $q_2 = 50$. In that case, their best response was $q_1 = 20$.

$$(20, 50)$$

$q_1 = 20$ is a best response against $q_2 = 50$ but is $q_2 = 50$ a best response against $q_1 = 20$?

Le's find out:

$$q_2 = 45 - \frac{1}{2}(20) = 35$$

Firm 2 has an incentive to change their strategy since 50 is not a best a best response to 20. These strategies are not *mututal* best responses.

Nash equilibrium: A set of strategies that are mututal best responses.

$(20, 50)$ is a nash equilibrium. What is the set of quantities that are mututal best responses:

$$q_1 = 45 - \frac{1}{2}q_2, q_2 = 45 - \frac{1}{2}q_1$$

If we solve this system of equations, we get a q_1, q_2 that are mutual best responses:

$$\{\{q_1 \to 30, q_2 \to 30\}\}$$

If we both choose a quantity of 30, then I am best responding to you and you are best-responding to me.