1 Production (Chapter 13)

Technology: turning inputs into outputs.

1.1 Input Bundles

(x_1, x_2) - x_1 apples, x_2 crusts.

$(2, 1)$ 2 apples and 1 crust.

$(4, 2)$ 4 apples and 2 crusts.

1.2 Technology / Production Function

Way of turning input bundles into *outputs.*

Describe a technology that turns 2 apples and 1 crust into a pie.

Apples and Crusts are the Inputs

Pies is the Output.

$(-2, -1, 1)$ is a element of the technology.

$(-4, -2, 2)$ is element of the technology (producting 2 pies).

Production is a way of describing a technology where the inputs are never outputs and the output is never an input.

And futhermore, there is only one output.

$f(x_1, x_2)$- **Production Function**

Maps an input bundle into the amount of output.

The production function for pies:

$$f(x_1, x_2) = min\left\{\frac{1}{2}x_1, x_2\right\}$$

$$f(2, 1) = min\left\{\frac{1}{2}(2), 1\right\} = min\{1, 1\} = 1$$

$$f(4, 2) = min\left\{\frac{1}{2}(4), 2\right\} = min\{2, 2\} = 2$$

As a utility function $min\left\{\frac{1}{2}x_1, x_2\right\}$ and $2 * min\left\{\frac{1}{2}x_1, x_2\right\}$ represent the same preferences. They are interchangable.

Production functions have a meaninful number as their output.

As a production function $min\left\{\frac{1}{2}x_1, x_2\right\}$ and $2 * min\left\{\frac{1}{2}x_1, x_2\right\}$ represent different technology. The later is more productive.

Production functions are **cardinal.** We cannot transform them into other production functions.

1.2.1 Example Perfect Complements

$f(x_1, x_2) = min\left\{\frac{1}{2}x_1, x_2\right\}$ use 2 apples and 1 crust to make a pie.

1.2.2 Example Cobb Douglass

You can use either human labor x_1 or tools x_2 to produce some good.

Too many humans and no tools produces few outputs.

Too many tools and no humans produces few outputs.

$f(x_1, x_2) = x_1 x_2$

$f(0, 10) = 0 * 10 = 0$

$f(10, 0) = 10 * 0 = 0$

$f(5, 5) = 5 * 5 = 25$

Here, you needs both some tools and some human labor to produce output.

1.3 Isoquants

Isoquant is defined as a set of input bundles that produce the same amount of output. (Different recipes for producing a desired level of output.)

$f(x_1, x_2) = x_1 x_2$

What are some input bundles that all produce output 25?

$$f(25, 1) = 25, f(1, 25) = 25, f(5, 5) = 25$$

$(25, 1), (1, 25), (5, 5)$ are on the same **iso**quant.

Isoquants are analagous to consumer indifference curves.

1.4 Marginal Products

Marginal product is how much extra output a firm will get by increasing **one** input a little while holding the other's fixed.

Analogous to marginal utility for a consumer. MUs were only really used for producing MRS. $MRS = -\frac{mu_1}{mu_2}$

Suppose $MP_1 = 1$ this says the extra output the firm will get by increasing input 1 by one unit is 1.

Suppose $MP_1 = 2$ this says the extra output the firm will get by increasing input 1 by one unit is 2.

$$MP_1 = \frac{\partial (f(x_1, x_2))}{\partial x_1}$$

$$MP_2 = \frac{\partial (f(x_1, x_2))}{\partial x_2}$$

1.4.1 Example Cobb Douglass

$f(x_1, x_2) = x_1 x_2$

A) What are the marginal products of this production function?

$$MP_1 = \frac{\partial (x_1 x_2)}{\partial x_1} = x_2$$

$$MP_2 = x_1$$

B) How much extra output (approximately) will the firm get if it is currently using input bundle $(2, 2)$. If it increases input 1 by one unit?

$$MP_1 = 2$$

For every unit the firm increases x_1 by, it will two extra units of output.

$$f(2, 2) = 4$$

$$f(3, 2) = 6$$

$$f(4, 2) = 8$$

$$f(5, 2) = 10$$

B) How much extra output (approximately) will the firm get if it is currently using input bundle $(3, 3)$. If it increases input 1 by one unit?

$$MP_1 = 3$$

$$f(3, 3) = 9, f(4, 3) = 12$$

Marignal product measures the productivity of each input in isolation.

1.5 Diminishing Marginal Product

Diminishing marginal product is the notion that an input will become less and less effctive as we increase it in isolation.

A production that has this property is said to have diminishing marginal product.

To check this, ask:

"is MP_1" decreasing as we increase x_1?"

"is MP_2" decreasing as we increase x_2?"

$$f(x_1, x_2) = x_1 x_2, MP_1 = x_2, MP_2 = x_1$$

$$f(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$$

$$MP_1 = \frac{\partial \left(x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} \right)}{\partial x_1} = \frac{1}{2} x_1^{-\frac{1}{2}} x_2^{\frac{1}{2}} = \frac{1}{2} \frac{x_2^{\frac{1}{2}}}{x_1^{\frac{1}{2}}}$$

$$MP_2 = \frac{\partial \left(x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} \right)}{\partial x_2} = \frac{1}{2} x_2^{-\frac{1}{2}} x_1^{\frac{1}{2}} = \frac{1}{2} \frac{x_1^{\frac{1}{2}}}{x_2^{\frac{1}{2}}}$$

This production function has decreasing marginal product with respect to input 1 and input 2.

At $(4, 4)$ $MP_1 = \frac{1}{2} \frac{2}{2} = \frac{1}{2} = 0.5$

At $(5, 4)$ $MP_1 = \frac{1}{2} \frac{\sqrt{4}}{\sqrt{5.0}} = 0.447214$

As you scale x_1, additional units of x_1 become less and less effective at producing output.

1.6 Example

a) Find the marginal products for the following production functions. Which has diminishing marginal products for both inputs?

a) $x_1 + x_2$ (perfect substitutes production)

$$mp_1 = \frac{\partial (x_1 + x_2)}{\partial x_1} = 1, mp_2 = \frac{\partial (x_1 + x_2)}{\partial x_2} = 1$$

Constant marginal products.

b) $x_1^{\frac{1}{2}} + x_2^{\frac{1}{2}}$

$f(4,4) = 2 + 2 = 4$

$f(5,4) = \sqrt{5.0} + 2 = 4.23607$

$f(6,4) = \sqrt{6.0} + 2 = 4.44949$

$$mp_1 = \frac{\partial \left(x_1^{\frac{1}{2}} + x_2^{\frac{1}{2}}\right)}{\partial x_1} = \frac{1}{2} x_1^{-\frac{1}{2}} = \frac{1}{2}\frac{1}{x_1^{\frac{1}{2}}}$$

Diminishing marginal product for input 1.

$$mp_2 = \frac{\partial \left(x_1^{\frac{1}{2}} + x_2^{\frac{1}{2}}\right)}{\partial x_2} = \frac{1}{2} x_2^{-\frac{1}{2}} = \frac{1}{2}\frac{1}{x_2^{\frac{1}{2}}}$$

Diminishing marginal product for input 2.

1.6.1 Technical Rate of Substitution

Suppose x_1 is twice as productive as x_2. $mp_1 = 2$, $mp_2 = 1$

If I increase by use of input 1 by one unit, how much I decrease my use of input 2 so I continute to produce the same output?

I can decrease x_2 by two units. The slope of the isoquant is -2.

The **technical rate of substitution** (TRS)

$$TRS = -\frac{mp_1}{mp_2}$$

This measurs the slope of isoquant at a particular point and can be interpreted as how much you can decrease x_2 by if you increase x_1 by one unit.

This is analagous to $MRS = -\frac{mu_1}{mu_2}$.

1.7 Examples

$f(x_1, x_2) = x_1 + x_2$,$mp_1 = 1, mp_2 = 1$

$$TRS = -\frac{1}{1} = -1$$

$f(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$, $mp_1 = \frac{1}{2}\frac{x_2^{\frac{1}{2}}}{x_1^{\frac{1}{2}}}, mp_2 = \frac{1}{2}\frac{x_1^{\frac{1}{2}}}{x_2^{\frac{1}{2}}}$

$$TRS = -\frac{\frac{1}{2}\frac{x_2^{\frac{1}{2}}}{x_1^{\frac{1}{2}}}}{\frac{1}{2}\frac{x_1^{\frac{1}{2}}}{x_2^{\frac{1}{2}}}} = -\frac{1}{2}\frac{x_2^{\frac{1}{2}}}{x_1^{\frac{1}{2}}}\left(2\frac{x_2^{\frac{1}{2}}}{x_1^{\frac{1}{2}}}\right) = -\frac{x_2}{x_1}$$

$f(x_1, x_2) = x_1^{\frac{1}{2}} + x_2^{\frac{1}{2}}, mp_1 = \frac{1}{2}\frac{1}{x_1^{\frac{1}{2}}}, mp_2 = \frac{1}{2}\frac{1}{x_2^{\frac{1}{2}}}$

$$TRS = -\frac{\frac{1}{2}\frac{1}{x_1^{\frac{1}{2}}}}{\frac{1}{2}\frac{1}{x_2^{\frac{1}{2}}}} = \frac{1}{2}\frac{1}{x_1^{\frac{1}{2}}}\left(2x_2^{\frac{1}{2}}\right) = -\frac{\sqrt{x_2}}{\sqrt{x_1}}$$

1.8 Returns to Scale

1.9 Short vs Long Run Production