1 Production

1.1 What can a firm do?

Technology.

A technology that turns 2 apples and 1 crust into a pie.

(a, c, p)

$$T = \{(-2, -1, 1), (-4, -2, 2), \}$$

Here the apples and crusts are only ever inputs and there is only one kind of output (pies).

This is a simple enough technology that we can instead use a **production function** to describe this technology.

f is a function that *map* input bundles $(2, 1)$ into an amount of output.

$$f(2, 1) = 1, f(4, 2) = 2$$

If x_1 is apples and x_2 is crusts:

$$f(x_1, x_2) = min\left\{\frac{1}{2}x_1, x_2\right\}$$

Describe the technology that produces output using x_1 left handed workers and x_2 right handed workers. Perfect substitutes production function.

$$f(x_1, x_2) = 10x_1 + 10x_2$$

Production functions are *almost* analgous to consumer utility functions.

The key difference is that the amount of output is a meaninful number.

$f(2, 1) = 1$ two apples and one crust produce **one pie**.

$$u(2, 1) = 1$$

Utility is not meaninful on its own, it is only meaninufl in comparison to other utilities.

$$u(2, 1) < u(4, 2)$$

$$u(2,1) = 2, u(4,2) = 4$$

$$min\left\{\frac{1}{2}x_1, x_2\right\}$$

$$2min\left\{\frac{1}{2}x_1, x_2\right\}$$

The key difference between production and utility is that prodction functions are **cardinal**. The number we get is meaningful.

1.2 Isoquants

An **isoquant** is a set of input bundles that all produce the same amount of output.

These are analgous to indifference curves.

$$f(x_1, x_2) = min\left\{\frac{1}{2}x_1, x_2\right\}$$

$$f(2,1) = 1, f(3,1) = 1, f(2,2) = 1, f(2,3) = 1$$

All these bundles are on the same isoquant.

Bundles on an isoquant are different ways of producing the same amount of output.

$$f(x_1, x_2) = 10x_1 + 10x_2$$

$$f(5,5) = 100, f(10,0) = 100, f(0,10) = 100, f(7,3) = 100$$

These are all different ways of producing 100 units of output.

1.3 Technical Rate of Substitution

Technical rate of substitution (TRS). **Slope of an isoquant.**

Approximately: *How much can I reduce my use of x_2 if I increase use of x_1 by one unit.*

TRS of the production function $f(x_1, x_2)$ is -1. You can give up one x_2 for every additional x_1 you use.

1.4 Marginal Product

Marginal product is a measure of productivity of an input.

Approximately: *How much extra output do I get by increasing one of the inputs by one unit?*

$f(5,5) = 100, f(6,5) = 110$

Technically the marginal product is the partial derivative of the production function with respect to the input.

$$MP_1 = \frac{f(x_1, x_2)}{\partial x_1} = \frac{10x_1 + 10x_2}{\partial x_1} = 10$$

$$MP_2 = \frac{f(x_1, x_2)}{\partial x_2} = \frac{10x_1 + 10x_2}{\partial x_2} = 10$$

1.5 TRS from MP

$$TRS = -\frac{MP_1}{MP_2}$$

(See how similar this is to $MRS = -\frac{MU_1}{MU_2}$)

Suppose $MP_1 = 2$ and $MP_2 = 1$.

TRS measures how much you can decrease x_2 by if you use one more unit of x_1.

$$TRS = -\frac{2}{1}$$

For every x_1 you can give up 2 x_2.

Suppose $MP_1 = 1$ and $MP_2 = 2$.

$$TRS = -\frac{1}{2}$$

1.6 Examples

For each, write the MP_1, MP_2,TRS.

a) $f(x_1, x_2) = x_1 + 2x_2$

$$MP_1 = \frac{\partial(x_1 + 2x_2)}{\partial x_1} = 1, MP_2 = \frac{\partial(x_1 + 2x_2)}{\partial x_2} = 2$$

$$TRS = -\frac{1}{2}$$

b) $f(x_1, x_2) = x_1 x_2$

$$MP_1 = \frac{\partial (x_1 x_2)}{\partial x_1} = x_2, MP_2 = \frac{\partial (x_1 x_2)}{\partial x_2} = x_1$$

$$TRS = -\frac{x_2}{x_1}$$

c) $f(x_1, x_2) = \left(x_1^{\frac{1}{2}} + x_2^{\frac{1}{2}}\right)$ (CES Production)

$$MP_1 = \frac{\partial \left(x_1^{\frac{1}{2}} + x_2^{\frac{1}{2}}\right)}{\partial x_1} = \frac{1}{2} x_1^{\frac{1}{2}-1} = \frac{1}{2} x_1^{-\frac{1}{2}} = \frac{1}{2} \frac{1}{\sqrt{x_1}}$$

Because x_1 only appears in the denominator, MP_1 is decreasing in x_1. Thus, the productivity of x_1 decreases the more that used.

Diminishing Marginal Product. The extra output you get from increasing one input decreases.

$$MP_2 = \frac{\partial \left(x_1^{\frac{1}{2}} + x_2^{\frac{1}{2}}\right)}{\partial x_2} = \frac{1}{2} x_2^{\frac{1}{2}-1} = \frac{1}{2} x_2^{-\frac{1}{2}} = \frac{1}{2} \frac{1}{\sqrt{x_2}}$$

$$TRS = -\frac{\frac{1}{2}\frac{1}{\sqrt{x_1}}}{\frac{1}{2}\frac{1}{\sqrt{x_2}}} = -\frac{\frac{1}{\sqrt{x_1}}}{\frac{1}{\sqrt{x_2}}} = -\frac{\frac{1}{\sqrt{x_1}}}{\frac{1}{\sqrt{x_2}}} = -\frac{1}{\sqrt{x_1}} \frac{\sqrt{x_2}}{1} = -\sqrt{\frac{x_2}{x_1}}$$

What is the TRS for each prodction function at the bundle $(4, 1)$?

$-\frac{1}{2}, -\frac{1}{4}, -\frac{1}{2}$