https://gecon.shinyapps.io/distopiaai/

1 Cost Min

When a firm is maximizing profit, they produce some output y.

That output is made by some combination of inputs (x_1, x_2) that prodcues that output $f(x_1, x_2) = y$.

Revenue does not depend on how they produce the output.

If you can produce the same output in a cheaper way, you can increase profit since revenue remains the same but costs decrease.

Profit maximization implies cost minimiztion.

1.1 Two-Step Process

(Today) Cost-Minimization: Find the cheapest way to produce any level of output y.

Profit-Maximization: Find the optimal y.

1.2 The Goal

Consumers: Find the (x_1, x_2) that maximize $u(x_1, x_2)$ subject to $p_1x_2 + p_2x_2 = m$

The opposite of the consumer problem (the dual):

Dual Consumers: Find the (x_1, x_2) that minimizes the cost $p_1x_1 + p_2x_2$ of achieving $u(x_1, x_2) = \bar{u}$.

Producer Cost-Min: Find the (x_1, x_2) that minimizes $w_1x_1 + w_2x_2$ subject to $f(x_1, x_2) = y$

Mathmatically the producer problem is equivelent to the dual consumer problem. Mathematically, they have the same properties.

1.3 Isocosts and Isoquants

An **isoquant** is a set of bundles that produce the same output $f(x_1, x_2) = y$

An **isocost** is a set of bundles that cost the same $w_1x_1 + w_2x_2 = c$

Geometrically the firm's problem is to find the bundle on the isoquant that is on the lowers isoquant.

1.4 Three Possibilities

If the utility function is "smooth" (you can take it's partial derivative):

Tangency Condition. The slope of the isoquant has to the be the same as the slope of the isocost.

$$TRS = -\frac{w_1}{w_2}$$

$$-\frac{MP_1}{MP_2} = -\frac{w_1}{w_2}$$

If we have perfect complments production, the TRS is not well-defined.

no-waste condition

For example if $f(x_1, x_2) = min\{x_1, x_2\}$

$$x_1 = x_2$$

If we have perfect substitutes production, the TRS is well-defined but just a number. If we check the tangency condition we might get something like:

One of the two endpoints (use all x_1 or use all x_2) will be optimal. Figure whether using all x_1 or all x_2 to produce y is cheapest.

For example:

$f(x_1, x_2) = 2x_1 + x_2$ $w_1 = 1, w_2 = 1$. Use all x_1.

1.4.1 Interpreting the Tangency Condition

The tangency condition for producers:

$$-\frac{MP_1}{MP_2} = -\frac{w_1}{w_2}$$

$$\frac{MP_1}{w_1} = \frac{MP_2}{w_2}$$

$\frac{MP_i}{p_i}$ is the extra output I get by spending one more dollar on input i

$$\frac{w_2}{MP_2} = \frac{w_1}{MP_1}$$

The cost of increasing output by 1 unit using each input is the same.

$w_2 = 2, MP_2 = 1$ what the cost of increains output by 1 unit using x_2?

If those are not the same:

$$\frac{w_2}{MP_2} > \frac{w_1}{MP_1}$$

In this case, if I reduce my use of x_2 but increase my use of x_1 so that the production remains the same I save more money $\frac{w_2}{MP_2}$ than I incur $\frac{w_1}{MP_1}$ and I still produce y. This is not optimal.

1.4.2 An Example

$w_1 = 1, w_2 = 1$

$f(x_1, x_2) = 2x_1 + x_2$

Since this is a perfect substitutes production function, one of the two extremes will be optimal. Use only x_1 or use only x_2.

To produce output y:

The cost of using only x_1? $f(x_1, 0) = 2x_1 + 0$. $2x_1 = y$. $x_1 = \frac{y}{2}$. The cost of this is $\frac{y}{2}$.

The cost of using only x_2? $f(0, x_2) = 0 + x_2$. $x_2 = y$. The cost of this is y.

Since x_1 is cheaper, use the bundle $\left(\frac{y}{2}, 0\right)$.

Conditional Factor Demand: $x_1 = \frac{y}{2}, x_2 = 0$.

Conditional factor demand tells us how much of each input to use to produce y units of output in the cheapest way.

Cost Function: $c(y) = w_1 \frac{y}{2} + w_2 0 = \frac{y}{2}$

Cost function tells us the cost of the cheapest way of producing y.

$$\left(\frac{y}{2}, 0\right), c(y) = \frac{y}{2}$$

1.4.3 You Try #1

What are the conditional factor demands for producing output y. What is the cost function?

$w_1 = 1, w_2 = 1$

$f(x_1, x_2) = min\left\{\frac{1}{2}x_1, x_2\right\}$

No Waste:

$$\frac{1}{2}x_1 = x_2$$

Production constraint:

$$min\left\{\frac{1}{2}x_1, x_2\right\} = y$$

Solve these to get the conditional factor demand:

$$min\left\{x_2, x_2\right\} = y$$

$$x_2 = y$$

Plug this back into the no-waste condition:

$$\frac{1}{2}x_1 = y$$

$$x_2 = 2y$$

Optimal bundle for producing y (condition-factor demands)

$$(2y, y)$$

To find the cost function, calculate the cost of this bundle:

$$c\left(y\right) = w_1 2y + w_2 y$$

$$c\left(y\right) = 1 * 2y + 1 * y = 3y$$

1.4.4 You Try #2

$w_1 = 1, w_2 = 1$

$f\left(x_1, x_2\right) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$

Tangency Condition

$$-\frac{x_2}{x_1} = -\frac{1}{1}$$

Production Constraint:

$$x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} = y$$

What are the conditional factor demands. What is the cost function?

$$x_2 = x_1$$

Plug that into the production constraint:

$$x_1^{\frac{1}{2}} x_1^{\frac{1}{2}} = y$$

$$x_1 = y$$

Plug this into the tangency condition:

$$y = x_2$$

Conditional factor demands:

$$x_1 = y, x_2 = y$$

How much output does this budle of inputs (y, y) actually produce?

$$f(y, y) = y^{\frac{1}{2}} y^{\frac{1}{2}} = y$$

Cost function. Calculate the cost of bundle (y, y)

$$w_1 y + w_2 y = 1y + 1y = 2y$$

$$c(y) = 2y$$

1.5 One More Example

$f(x_1, x_2) = x_1^{\frac{1}{4}} x_2^{\frac{1}{4}}$, $w_1 = 1, w_2 = 1$

Tangency:

$$x_1 = x_2$$

Production constraint:

$$x_1^{\frac{1}{4}} x_2^{\frac{1}{4}} = y$$

Plug tangency into production:

$$x_1^{\frac{1}{4}} x_1^{\frac{1}{4}} = y$$

$$x_1^{\frac{1}{2}} = y$$

$$x_1 = y^2, x_2 = y^2$$

$$c(y) = 2y^2$$

$$c(2) = 2 * 4 = 8$$

$$c(4) = 2 * 16 = 32$$

1.5.1 Short Run Cost

We say a company is in the "short-run" if one of the inputs is fixed.

$$f(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$$

Conditional factor demands: $x_1 = y, x_2 = y, c(y) = 2y$

In the short run, suppose $x_2 = 4$.

What is the cost function?

Given that $x_2 = 4$?

Short run production is:

$$f(x_1, 4) = x_1^{\frac{1}{2}} 4^{\frac{1}{2}}$$

How much x_1 do I need to produce y:

$$x_1^{\frac{1}{2}} 4^{\frac{1}{2}} = y$$

$$x_1^{\frac{1}{2}} 2 = y$$

$$x_1^{\frac{1}{2}} = \frac{1}{2} y$$

$$x_1 = \frac{1}{4} y^2$$

The cost of producing y in the short run is the cost of the bundle $\left(\frac{1}{4}y, 4\right)$

$$w_1 \frac{1}{4} y^2 + w_2 4 = \frac{1}{4} y^2 + 4$$

$$c_{sr}(y) = \frac{1}{4} y^2 + 4$$

When is the short run cost bigger than the long-run cost?

$$Reduce[\frac{1}{4} y^2 + 4 > 2y]$$

$$y < 4 \vee y > 4$$