1 Elasticity

$x_1 = \frac{\frac{1}{2}m}{p_1}$

$q = \frac{100}{p}$

Price elascitity

What is the price elasticity of demand

$$Q = \frac{100}{p}$$

$$\frac{\partial \left(\frac{100}{p}\right)}{\partial p} \frac{p}{\frac{100}{p}}$$

$$\frac{\partial \left(100p^{-1}\right)}{\partial p} = 100 \left(-1\right) p^{-2} = \frac{-100}{p^2}$$

$$\epsilon = \frac{-100}{p^2} \frac{p}{\frac{100}{p}} = \frac{-100}{p^2} \frac{p^2}{100} = -1$$

When the price is $p = 10$

$$\frac{-100}{100} \frac{10}{10} = -1$$

Elasticity is -1 which means when price goes up by 1% demand decreases by 1%.

cross-price elasticity

$x_1 = \frac{10}{p_1+p_2}$ what is the cross-price elasticity for x_1 with respect to p_2

$$\frac{\partial \left(\frac{10}{p_1+p_2}\right)}{\partial p_2} \frac{p_2}{\frac{10}{p_1+p_2}} = -\frac{p_2}{p_1 + p_2}$$

Income elasticity

How does demand change in percentage terms when income increases by 1%.

$$\frac{\partial \left(\frac{\frac{1}{2}}{p_1} m\right)}{\partial m} \frac{m}{\frac{\frac{1}{2}m}{p_1}} = \frac{\frac{1}{2}}{p_1} \frac{m}{\frac{\frac{1}{2}m}{p_1}} = \frac{\frac{1}{2}m}{p_1} \frac{1}{\frac{\frac{1}{2}m}{p_1}} = 1$$

as income increases by 1%, demand increases by 1%.

1.1 Classifying elasticity

Price elasticity:

$\epsilon < -1$ for example -10. Elastic Demand

$\epsilon > -1$ for example -0.01. Inealstic Demand

$\epsilon = -1$ Unit-elastic demand.

2 Equiblirbrium Problems

$Q_d = 200 - 40p$, $Q_s = 10p$

Equilibrium is a price p such that $Q_d = Q_s$

Set demand and supply function euqal:

$$200 - 40p = 10p$$

$$200 = 50p$$

$$p = 4,$$

To get qunaity, plug back into either demand of supply:

$$Q = 40$$

b) Elasticity of demand at the equilibrium price:

$$\frac{\partial (200 - 40p)}{\partial p} \frac{p}{200 - 40p}$$

$$-40 \frac{p}{200 - 40p}$$

Plug in the equilibrium price:

$$-40\frac{4}{200-40\,(4)} = -40\frac{4}{40} = -4$$

c) Find the equilibrium with $t = \frac{5}{2}$

$$200 - 40\left(p + \frac{5}{2}\right) = 10p$$

$$200 - 40\left(p + \frac{5}{2}\right) = 10p$$

$$200 - 40p - 100 = 10p$$

$$100 = 50p$$

$$p = 2$$

If we plug this back into the demand to find quantity, make sure to add the tax.

$$Q_d = 200 - 40\left(2 + \frac{5}{2}\right) = 200 - 40 - 100 = 20$$

$$Q_s = 10\,(2) = 20$$

$$Q = 20$$

d) Deadweight loss is always $t\,(q^* - q^*_{tax}) = \frac{5}{2}\,(40-20) = \frac{5}{2}20 = 50$

3 Cost Min

Where can a cost-minimizing bundle occur?

1) Tangency

$$TRS = -\frac{w_1}{w_2}$$

If this can be true at the optimum it has to be.

2) Perfect Substitutes Production

$f(x_1, x_2) = 2x_1 + x_2$ $w_1 = 1, w_2 = 1$

$$TRS = -\frac{w_1}{w_2}$$

$$-2 = -1$$

The tangency condition can **never** be true.

One of the two endpoints of the production constraint will be optimal.

Use either all x_1 or all x_2 to produce y. Just figure out which is cheaper.

In the case above, using all x_1 is optimal.

$$2x_1 = y$$

Conditional Factor demand:

$$x_1 = \frac{1}{2}y$$

$c(y) = (1)\frac{1}{2}y + (1)0 = \frac{1}{2}y$

3) There is no slope of the isoquant.

$$f(x_1, x_2) = min\{x_1, x_2\}$$

No-waste condition:

$$x_1 = x_2$$

Conditional factor demand:

Plug the no-waste condition into the production constraint:

$$y = min\left\{x_1, x_2\right\}$$

$$y = min\left\{x_1, x_1\right\} = x_1$$

$$x_1 = y$$

$$x_2 = y$$

$c\left(\right) = w_1 y + w_2 y$

If $w_1 = 1, w_2 = 1$

$c\left(y\right) = y + y$

$$c\left(y\right) = 2y$$

Marginal cost:

$$\frac{\partial\left(2y\right)}{\partial y} = 2$$

This is constant marginal cost.

3.1 Short run costs.

Suppose $x_2 = 4$

$x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$

How much x_1 do I need to produce y if $x_2 = 4$.

$$f\left(x_1, 4\right) = x_1^{\frac{1}{2}} 4^{\frac{1}{2}}$$

$$2x_1^{\frac{1}{2}} = y$$

$$x_1^{\frac{1}{2}} = \frac{1}{2} y$$

$$x_1 = \frac{1}{4}y^2$$

Cost function:

$$c\left(y\right) = w_1 \frac{1}{4}y^2 + w_2 4$$

If $w_1 = w_2 = 1$

$$c\left(y\right) = \frac{1}{4}y^2 + 4$$

4 Profit

$$\pi\left(y\right) = rev\left(y\right) - c\left(y\right)$$

To maximize a profit function, we always look for the where the margainl profit $\pi'\left(y\right) = 0$

$$mr\left(y\right) = mc\left(y\right)$$

$$\pi\left(y\right) = p\left(y\right) * y - c\left(y\right)$$

Under price taking, we assume that price is fixed: $p\left(y\right) = p$

$$\pi\left(y\right) = py - c\left(y\right)$$

Specifically under price taking, $mr\left(y\right) = mc\left(y\right)$ simplifies to this condition:

$$p = mc\left(y\right)$$

$p = 10$ $c\left(y\right) = y^2$

$$\pi\left(y\right) = 10y - y^2$$

$$\pi'\left(y\right) = 10 - 2y$$

$$10 = 2y$$

$$y = 5$$

5 Monopoly

$c(y) = y^2$

We can no longer assume the price a monopolist gets is fixed. The price **is the** inverse demand.

Suppose demand is $y = 100 - p$

a) What is the inverse demand?

$$y = 100 - p$$

$$p = 100 - y$$

b)

$$\pi(y) = (100 - y)\, y - y^2$$

$$100y - y^2 - y^2$$

$$\pi(y) = 100y - 2y^2$$

Marginal profit:

$$100 - 4y = 0$$

$$y = 25$$

What price do they charge?

Plug this back into the inverse demand:

$$p = (100 - 25) = 75$$

6 Profit in cournot

Inverse demand is $q = 500 - p$

$p = 500 - q$

$$q_1 \left(500 - (q_1 + q_2)\right) - 200q_1$$