Solutions Problem Set 7

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1 Solutions for Chapter 18

Solution for Exercise 1.

a) Plug endowments into demands

$$x_{1,a} = \frac{\frac{1}{2}(20p_1)}{p_1}, x_{2,a} = \frac{\frac{1}{2}(20p_1)}{p_2}$$

$$x_{1,b} = \frac{20p_2}{p_1 + p_2}, x_{2,b} = \frac{20p_2}{p_1 + p_2}$$

Market clearing for good 1:

$$\frac{\frac{1}{2}(20p_1)}{p_1} + \frac{20p_2}{p_1 + p_2} = 20$$

Market clearing for good 2:

$$\frac{\frac{1}{2}(20p_1)}{p_2} + \frac{20p_2}{p_1 + p_2} = 20$$

b) No, the markets don't clear:

$$\frac{\frac{1}{2}(20(1))}{1} + \frac{20(2)}{1+2} = 23.3333 \neq 20$$

$$\frac{\frac{1}{2}(20(1))}{2} + \frac{20(2)}{1+2} = 18.3333 \neq 20$$

c) Solve either market clearing condition for p_2. Let's use market 1:

$$\frac{\frac{1}{2}(20(1))}{1} + \frac{20p_2}{1 + p_2} = 20$$

$$p_2 = 1$$

d) Plug in price $p_1 = 1, p_2 = 1$ into demands:

$$x_{1,a} = 10, x_{2,a} = 10$$

$$x_{1,b} = 10, x_{2,b} = 10$$

e) No, the first welfare theorem tells us this will be pareto efficient.

Solution for Exercise 2.

a)

Plug endowments into demands

$$x_{1,a} = \frac{\frac{1}{2}\left(20p_1\right)}{p_1}, x_{2,a} = \frac{\frac{1}{2}\left(20p_1\right)}{p_2}$$

$$x_{1,b} = \frac{\frac{1}{2}\left(20p_2\right)}{p_1}, x_{2,b} = \frac{\frac{1}{2}\left(20p_2\right)}{p_2}$$

Market clearing for good 1:

$$\frac{\frac{1}{2}\left(20p_1\right)}{p_1} + \frac{\frac{1}{2}\left(20p_2\right)}{p_1} = 20$$

Market clearing for good 2:

$$\frac{\frac{1}{2}\left(20p_1\right)}{p_2} + \frac{\frac{1}{2}\left(20p_2\right)}{p_2} = 20$$

c) Solve either market clearing condition for p_2. Let's use market 1:

$$\frac{\frac{1}{2}\left(20\left(1\right)\right)}{1} + \frac{\frac{1}{2}\left(20p_2\right)}{1} = 20$$

$$p_2 = 1$$

d) Plug in price $p_1 = 1, p_2 = 1$ into demands:

$$x_{1,a} = 10, x_{2,a} = 10$$

$$x_{1,b} = 10, x_{2,b} = 10$$

e) No, the first welfare theorem tells us this will be pareto efficient.

Solution for Exercise 3.

a)

Plug endowments into demands

$$x_{1,a} = \frac{\frac{1}{2}(10p_1)}{p_1}, x_{2,a} = \frac{\frac{1}{2}(10p_1)}{p_2}$$

$$x_{1,b} = \frac{\frac{1}{2}(20p_2)}{p_1}, x_{2,b} = \frac{\frac{1}{2}(20p_2)}{p_2}$$

Market clearing for good 1:

$$\frac{\frac{1}{2}(10p_1)}{p_1} + \frac{\frac{1}{2}(20p_2)}{p_1} = 10$$

Market clearing for good 2:

$$\frac{\frac{1}{2}(10p_1)}{p_2} + \frac{\frac{1}{2}(20p_2)}{p_2} = 20$$

Solve either market clearing condition for p_2. Let's use market 1:

$$\frac{\frac{1}{2}(10(1))}{1} + \frac{\frac{1}{2}(20p_2)}{1} = 10$$

$$p_2 = \frac{1}{2}$$

Solution for Exercise 4.

a)

Plug endowments into demands

$$x_{1,a} = \frac{\frac{1}{2}(20p_1)}{p_1}, x_{2,a} = \frac{\frac{1}{2}(20p_1)}{p_2}$$

$$x_{1,b} = \frac{\frac{2}{3}(20p_2)}{p_1}, x_{2,b} = \frac{\frac{1}{3}(20p_2)}{p_2}$$

Market clearing for good 1:

$$\frac{\frac{1}{2}(20p_1)}{p_1} + \frac{\frac{2}{3}(20p_2)}{p_1} = 20$$

Market clearing for good 2:

$$\frac{\frac{1}{2}(20p_1)}{p_2} + \frac{\frac{1}{3}(20p_2)}{p_2} = 20$$

b)

Solve either market clearing condition for p_2. Let's use market 1:

$$\frac{\frac{1}{2}(20(1))}{1} + \frac{\frac{2}{3}(20p_2)}{1} = 20$$

$$p_2 = \frac{3}{4}$$

c)

$$x_{1,a} = 10, x_{2,a} = \frac{40}{3}$$

$$x_{1,b} = 10, x_{2,b} = \frac{20}{3}$$