Interpretation of the Derivative

If $y = f(x)$ then,

1. $m = f'(a)$ is the slope of the tangent line to $y = f(x)$ at $x = a$ and the equation of the tangent line at $x = a$ is given by $y = f(a) + f'(a)(x-a)$.
2. $f'(a)$ is the instantaneous rate of change of $f(x)$ at $x = a$.

Basic Properties and Formulas

If $f(x)$ and $g(x)$ are differentiable functions (the derivative exists), c and n are any real numbers,

1. $\frac{d}{dx}\left(c\right) = 0$
2. $\left(c\,f(x)\right)' = c\,f'(x)$
3. $\frac{d}{dx}\left(x^n\right) = n\,x^{n-1}$ – **Power Rule**
4. $\left(f(x) \pm g(x)\right)' = f'(x) \pm g'(x)$
5. $\left(f(x)\,g(x)\right)' = f'(x)\,g(x) + f(x)\,g'(x)$ – **Product Rule**
6. $\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)\,g(x) - f(x)\,g'(x)}{\left(g(x)\right)^2}$ – **Quotient Rule**
7. $\frac{d}{dx}\left(f\left(g(x)\right)\right) = f'\left(g(x)\right)g'(x)$ – **Chain Rule**

Common Derivatives

$$\frac{d}{dx}\left(x\right) = 1$$

$$\frac{d}{dx}\left(\mathbf{e}^x\right) = \mathbf{e}^x$$

$$\frac{d}{dx}\left(\ln(x)\right) = \frac{1}{x},\quad x > 0$$

Partial Derivatives

Partial Derivatives are simply holding all other variables constant (and act like constants for the derivative) and only taking the derivative with respect to a given variable.

1. If $z = f(x, y) = x^4y^3 + 8x^2y + y^4 + 5x$, then the partial derivatives are

$$\frac{\partial z}{\partial x} = 4x^3y^3 + 16xy + 5 \qquad \text{(Note: } y \text{ fixed, } x \text{ independent variable, } z \text{ dependent variable)}$$

$$\frac{\partial z}{\partial y} = 3x^4y^2 + 8x^2 + 4y^3 \qquad \text{(Note: } x \text{ fixed, } y \text{ independent variable, } z \text{ dependent variable)}$$

2. If $z = f(x, y) = (x^2 + y^3)^{10} + \ln(x)$, then the partial derivatives are

$$\frac{\partial z}{\partial x} = 20x(x^2 + y^3)^9 + \frac{1}{x} \qquad \text{(Note: We used the chain rule on the first term)}$$

$$\frac{\partial z}{\partial y} = 30y^2(x^2 + y^3)^9 \qquad \text{(Note: Chain rule again, and second term has no } y\text{)}$$