1 Social Choice Functions

Preference Aggregation Rule - turns the set of individual preference $\succsim_i$ into a social preference $\succsim^*$.

Social Choice Functions - turns the set of individual preferences $\succsim_i$ into a choice for society (from the set of all outcomes).

1.1 Social Choice from Preference Aggregation

Since any complete and transitive preference aggregation rule creates a complete and transitive social preference relation, there is some "best" or set of "best" outcomes according to that preference relation which are the choices it would make from the set of all outcomes.

1.1.1 Dictatorship- Social Choice Function

Let's assume 1 is the dictator.

The **dictator social choice** function picks a choice that is the favorite outcome of the dictator.

Example 1:

$1 : a \succ b \succ c$

$2 : a \succ c \succ b$

$3 : c \succ a \succ b$

Dictatorship the Preference Aggregation Rule results in **social preference relation-** $a \succ^* b \succ^* c$**.**

Dictatorship the Social Choice Function results in **choice** a**.**

Example 2:

$1 : a \succ c \succ b$

$2 : a \succ c \succ b$

$3 : b \succ c \succ a$

$4 : b \succ a \succ c$

$5 : c \succ a \succ b$

Dictatorship the Preference Aggregation Rule results in **social preference relation-** $a \succ^* c \succ^* b$**.**

Dictatorship the Social Choice Function results in **choice** a**.**

1.1.2 Unanimity- The Social Choice Function

An outcome is chosen as the *social choice* if that outcome is unanimously better than every other outcome.

Example 1:

$1 : a \succ b \succ c$

$2 : a \succ c \succ b$

$3 : c \succ a \succ b$

This social choice function does not pick a choice.

Example 2:

$1 : a \succ c \succ b$

$2 : a \succ c \succ b$

$3 : b \succ c \succ a$

$4 : b \succ a \succ c$

$5 : c \succ a \succ b$

This social choice function does not pick a choice.

Example 3:

$1 : a \succ c \succ b \succ d$

$2 : a \succ c \succ d \succ b$

$3 : a \succ d \succ b \succ c$

a is chosen as the social choice.

1.1.3 Plurality

The social choice is the outcome that is ranked first by the most people. (It gets the most votes).

Example 1:

$1 : a \succ b \succ c$

$2 : a \succ c \succ b$

$3 : c \succ a \succ b$

a:2,c:1,b:0

The social choice is a.

Example 2:

$1 : a \succ c \succ b$

$2 : a \succ c \succ b$

$3 : b \succ c \succ a$

$4 : b \succ a \succ c$

$5 : c \succ a \succ b$

$a : 2, b : 2, c : 1$

a and b are the social choices.

1.1.4 Veto

Minimize the number of people who get their least favorite. Social choice is the outcome who the fewest number of people rank last.

Example 1:

$1 : a \succ b \succ c$

$2 : a \succ c \succ b$

$3 : c \succ a \succ b$

$a : 0, b : 2, c : 1$

The social choice is a.

Example 2:

$1 : a \succ c \succ b$

$2 : a \succ c \succ b$

$3 : b \succ c \succ a$

$4 : b \succ a \succ c$

$5 : c \succ a \succ b$

$a : 1, b : 3, c : 1$

a and c are the social choices.

1.1.5 Borda Count

The social choice is the outcome that gets the highest Borda score.

Example 1:

$1 : a \succ b \succ c$

$2 : a \succ c \succ b$

$3 : c \succ a \succ b$

$a : 3 + 3 + 2 = 8$

$b : 2 + 1 + 1 = 4$

$c : 1 + 2 + 3 = 6$

a is the social choice.

Example 2:

$1 : a \succ c \succ b$

$2 : a \succ c \succ b$

$3 : b \succ c \succ a$

$4 : b \succ a \succ c$

$5 : c \succ a \succ b$

$a : 3 + 3 + 1 + 2 + 2 = 11$

$b : 1 + 1 + 3 + 3 + 1 = 9$

$c : 2 + 2 + 2 + 1 + 3 = 10$

a is social choice.

1.2 Tournament-Style Social Choice

Put all the outcomes into a tournament bracket. Take a pairwise vote over all pairs that play eachother in the tournament. The winner is the outcome that wins the tournament.

$1 : a \succ d \succ c \succ b$

$2 : d \succ a \succ c \succ b$

$3 : b \succ c \succ a \succ d$

$4 : b \succ d \succ a \succ c$

$5 : c \succ d \succ a \succ b$

a—_ ——c

b— ——d

In this tournament, a and b face eachother in a pair-wise vote. So do c and d. Then the winners of those votes face-off on another pairwise vote to determine the winner.

d is the social choice because it wins the tournament.

2 Properties of Social Choice Functions

2.1 Nonempty

A **nonempty** social choice function makes a choice for any set of individual preferences.

Unanimity is **not** nonempty since sometimes it results in no choice.

Every other social choice function we looked at above is **nonempty.**

2.2 Pareto Efficient

A social choice function is **pareto efficient** if anytime there is a y such that everyone agrees that some other outcome x is strictly better, then y should not be chosen.

Another way to think about this is that the social choice function **never** picks an outcome that is strictly pareto dominated.

2.2.1 Example of one that is not Pareto Efficient.

Suppose the social choice rule is a is always chosen.

$1 : b \succ a \succ c$

$2 : b \succ a \succ c$

$3 : b \succ a \succ c$

Since b strictly pareto dominates a, and a is chosen by this rule, it is not Pareto efficient.

2.3 IIA

A social choice function is **IIA** if for every two sets of preferences where the individual preferences are the same for two outcomes x and y.

If x is chosen from the first set and y was not then y cannot be chosen from the second.

2.4 Why IIA Matters for Social Choice

- Type 1 (25 People): a $\succ$ b $\succ$ c
- Type 2 (40 People): b $\succ$ c $\succ$ a
- Type 3 (35 People): c $\succ$ a $\succ$ b

$a : (25)\,3 + (40)\,(1) + (35)\,2 = 185$

$b : (25)\,2 + (40)\,3 + (35)\,1 = 205$

$c : (25)\,1 + (40)\,2 + (35)\,3 = 210$

c is the social choice

• Type 1 (25 People): b $\succ$ c

• Type 2 (40 People): b $\succ$ c

• Type 3 (35 People): c $\succ$ b

$b : (25)\,2 + (40)\,2 + (35)\,1 = 165$

$c : (25)\,1 + (40)\,1 + (35)\,2 = 135$

b is the social choice.

2.5 Classifying Social Choice Functions

2.5.1 Unanimity

Nonempty- $\times$ it sometimes results in an empty choice.

Pareto- If x is prefer ed by everyone to y then y cannot possibly be unanimously better than every other option, so y cannot be chosen.

IIA- If x is chosen as the outcome in the first set of preferences then it is unianmously better than every other option including y. In the second set, the preferences between x and y have to remain the same. x is still unanimously better than y. Since y cannot be unanimously better than x it cannot be the choice.

2.5.2 Plurality Vote

Nonempty- Because this is a rule that assigns a score (most votes) then some outcome(s) gets the highest score and is the choice.

Pareto-

As preference aggregation rule it is not Pareto efficient.

$a \succ b \succ c$

$a \succ b \succ c$

$a \succ b \succ c$

$a : 3, b : 0, c : 0.\ a \succ^* b \sim^* c$

As a social chocie function it **is** pareto efficient.

If x is unanimously better than y then y cannot be chosen.

If x is unanimously better than y then y cannot be anyone's favorite. y will get zero votes. Because at least some outcome gets at least one vote, then y cannot be chosen.

IIA-

Set 1:

$1 : a \succ b \succ c$

$2 : a \succ b \succ c$

$3 : b \succ a \succ c$

In set 1: $a : 2, b : 1, c : 0$. The winner (social choice) is a

Set 2:

$1 : a \succ b \succ c$

$2 : c \succ a \succ b$

$3 : b \succ a \succ c$

Each outcome gets 1 vote and the social choices are a, b, c.

Because we didn't change the prefrences between a and b. a was chosen in the first set but b wasn't. IIA says b should not be chosen in the second set, but it is.

Not IIA.

2.6 Arrow's Impossibility Round 2

Rule	Nonempty	Pareto	IIA
Dictator	✓	✓	✓
Unanimity	×	✓	✓
Plurality	✓	✓	×
Borda	✓	✓	×

Arrow's impossibility theorem also applies to choices. It says:

The only social choice function that is non-empty, pareto-efficient, and IIA is a dictatorship.