1 Cardinal Social Choice

1.1 Ordinal Models

$P = \{a, b\}$

$O = \{ab, a, b, n\}$

$b \succ_a ab \succ_a a \succ_a n$

$a \succ_b ab \succ_b b \succ_b n$

Borda Count

$b : 4 + 2,\ a : 2 + 4, ab : 3 + 3, n : 1 + 1$

$a \sim^* b \sim^* ab \succ^* n$

1.2 Cardinal Models

1.2.1 Cleaning

$P = \{a, b\}$

$O = \{ab, a, b, n\}$

$u_a(b) = 25, u_a(ab) = 12, u_a(a) = 10, u_a(n) = 5$

$u_b(b) = 10, u_b(ab) = 12, u_b(a) = 25, u_b(n) = 5$

1.3 Pareto Efficiency in Cardinal Models (Recap)

Pareto efficiency with ordinal preferences is:

x is Pareto efficient if nothing strictly Pareto dominates it.

A pareto efficient outcome is one where there is nothing *clearly better*.

x is Pareto efficient if there is no other outcome y such that everyone likes y at least as much and at least one person likes it strictly more.

There is no y such that $y \succsim_i x$ for everyone and for at least one person $y \succ_i x$.

In terms of utilities, x is Pareto efficient if there is no y such that:

$u_i(y) \geq u_i(x)$ for everyone

$u_i(y) > u_i(x)$ for at least one person.

1.3.1 Pareto efficient outcomes in Cleaning Example

$u_a(b) = 25, u_a(ab) = 12, u_a(a) = 10, u_a(n) = 5$

$u_b(b) = 10, u_b(ab) = 12, u_b(a) = 25, u_b(n) = 5$

n is not Pareto efficient. Any other outcome makes everyone strictly better off.

a, b, ab are Pareto efficient.

Pareto efficiency quickly reaches the end of its value. Here is has no way of comparing these three Pareto efficient outcomes.

1.4 More Clarification on Pareto

For everyone outcome ask this question: "can I make everyone at least as well of and at least one person strictly better off by picking some other outcome?" if the answer is **yes** the original outcome is not Pareto efficient.

1.5 Common Welfare Functions

A preference aggregation rule takes individual preferences and turns them into a preference for society.

A welfare function takes individual utilities and turns them into a utility for society 'welfare'.

$W(u_a, u_b)$

1.5.1 Utilitarian

Utilitarian welfare assigns welfare based on the total utility an outcome generates for society.

$W(u_a, u_b) = u_a + u_b$

$u_a(b) = 25, u_a(ab) = 12, u_a(a) = 10, u_a(n) = 5$

$u_b(b) = 10, u_b(ab) = 12, u_b(a) = 25, u_b(n) = 5$

Welfare of outcome a?

$W(10, 25) = 10 + 25 = 35$

Welfare of outcome b?

$W(25, 10) = 25 + 10 = 35$

Welfare of outcome ab?

$W(12, 12) = 12 + 12 = 24$

Welfare of outcome n?

$W(5, 5) = 5 + 5 = 10$

This induces the following social preferences:

$a \sim^* b \succ^* ab \succ^* n$

1.5.2 Rawlsian

$W\left(u_a, u_b\right) = min\left\{u_a, u_b\right\}$

Welfare of outcome a?

$W\left(10, 25\right) = 10$

Welfare of outcome b?

$W\left(25, 10\right) = 10$

Welfare of outcome ab?

$W\left(12, 12\right) = 12$

Welfare of outcome n?

$W\left(5, 5\right) = 5$

This induces the following social preferences:

$ab \succ^* a \sim^* b \succ^* n$

1.5.3 Nash Welfare Function

$W\left(u_a, u_b\right) = u_a^{\frac{1}{2}} u_b^{\frac{1}{2}}$

Welfare of outcome a?

$W\left(10, 25\right) = 10^{\frac{1}{2}} 25^{\frac{1}{2}} = \left(250\right)^{\frac{1}{2}}$

Welfare of outcome b?

$W\left(25, 10\right) = 25^{\frac{1}{2}} 10^{\frac{1}{2}} = \left(250\right)^{\frac{1}{2}}$

Welfare of outcome ab?

$W\left(12, 12\right) = 12^{\frac{1}{2}} 12^{\frac{1}{2}} = \left(144\right)^{\frac{1}{2}} = 144$

Welfare of outcome n?

$W\left(5, 5\right) = 5^{\frac{1}{2}} 5^{\frac{1}{2}} = \left(25\right)^{\frac{1}{2}} = 5$

$a \sim^* b \succ^* ab \succ^* n$