0.1 Cleaning Model

$P = \{a, b\}$

$O = \{ab, a, b, n\}$

$U_a(ab) = 12, U_a(b) = 25, U_a(a) = 10, U_a(n) = 5$

$U_b(ab) = 12, U_b(b) = 10, U_b(a) = 25, U_b(n) = 5$

0.1.1 Utilitarian Welfare

Utilitarian Welfare attempts to maximize the total utility in society. It a measure of *efficiency*. We often say that when an outcome maximizes utilitarian welfare that it **"maximizes efficiency"**.

$W(ab) = \frac{12+12}{2} = 12, W(a) = \frac{25+10}{2} = 17.5, W(b) = \frac{25+10}{2} = 17.5, W(n) = 5$

$$a \sim^* b \succ^* ab \succ^* n$$

a, b are the social choices induced by the utilitarian welfare function.

a, b "maximize efficiency"

0.1.2 Rawlsian Welfare

$W(ab) = min\{u_a(ab), u_b(ab)\} = min\{12, 12\} = 12$

$W(a) = min\{10, 25\} = 10, W(b) = 10, W(n) = 5$

$$ab \succ^* a \sim^* b \succ^* n$$

The social choice for Rawlsian welfare is ab.

The most fair outcome is ab.

Of the Pareto efficient outcomes, a and b are most efficient but ab is the most fair.

This creates a tension between efficiency and fairness.

0.1.3 Nash Welfare

$W(ab) = 12^{\frac{1}{2}} 12^{\frac{1}{2}} = 12$

$W(a) = 10^{\frac{1}{2}} 25^{\frac{1}{2}} = 15.8114$

$W(b) = 25^{\frac{1}{2}} 10^{\frac{1}{2}} = 15.8114$

$W(n) = 5$

$a \sim^* b \succ^* ab \succ^* n$

0.2 Coffee Machine Model

Public Goods Game

We have Alice and Bob. They need maintain the coffee machine. If no one maintains it, it works ok and the utility of both is \$10. If one maintains it, that is good for the other person but bad for them.

$P = \{a, b\}$

$O = \{ab, a, b, n\}$

$U_a(ab) = 15, U_a(a) = 5, U_a(b) = 20, U_n(n) = 10$

$U_b(ab) = 15, U_b(a) = 20, U_b(b) = 5, U_n(n) = 10$

Public Goods Game

0.3 Utilitarian

What is most efficient?

$W(ab) = \frac{15+15}{2} = 15$

$W(a) = \frac{5+20}{2} = 12.5$

$W(b) = \frac{20+5}{2} = 12.5$

$W(n) = \frac{10+10}{2} = 10$

$ab \succ^* a \sim^* b \succ^* n$

ab is most efficient.

0.3.1 Rawlsian Welfare

$W(ab) = 15$

$W(a) = 5$

$W(b) = 5$

$W(n) = 10$

$ab \succ^* n \succ^* a \sim^* b$

ab is most fair.