1 Homework Review

1.1 Pareto Efficiency Procedure

1.1.1 Pareto with Indifference

An outcome is Pareto efficient if there is no other outcome that makes everyone at least as well off and at least one person strictly better off.

For every outcome ask yourself "can I make everyone at least as well off and at least one person strictly better off?" If the answer is **yes** then the outcome **is not** Pareto efficient. If the answer is **no** the outcome is Pareto efficient.

1.2 Pareto Efficiency Example

$a \sim b \sim c \succ d$

$a \succ b \sim c \succ d$

d is not Pareto efficient.

c is not because of a

b is not because of a

a is Pareto efficient.

1.3 Pareto Efficiency Example

$b \sim c \succ d$

$b \sim c \succ d$

d is not because of c or b.

b and c are pareto efficient.

1.4 Pareto with Strict Preferences

What is mean to find something that is at least as good for everyone when preferences are strict?

When preferences of every individual are strict. An outcome is Pareto efficient if there is no other outcome that makes everyone strictly better off.

Ask yourself "Can I make everyone strictly better off?" if the answer is yes, the outcome is not Pareto efficient.

1.5 Example 4.4 from the notes.

a and b are Pareto efficient because that are someone's strict favorite. c because "no".

1.6 Pareto Efficiency Plurality Vote

Pareto efficiency for a preference aggregation rule requires that any time there are two outcomes b and c such that for everyone $b \succ_i c$ (b is unanimously better than c) then $b \succ^* c$.

Come up with an example of preferences for 3 individuals over outcomes a, b, c that shows that Plurality vote is not a Pareto efficient preference aggregation rule.

$a \succ b \succ c$

$a \succ b \succ c$

$a \succ b \succ c$

$a \succ^* b \sim^* c$

1.7 Pareto Efficiency Veto

$a \succ b \succ c$

$a \succ b \succ c$

$a \succ b \succ c$

$a \sim^* b \succ^* c$

1.8 Alphabetic Rule

The alphabetic rule orders the outcomes in terms of alphabetical order regardless of individual preferences. Come up with a counter-example showing it is not Pareto efficient.

1.9 Come up with a rule that is everything except Pareto efficient.

The alphabetic rule above is everything except Pareto efficient.

a, b, c $a \succ^* b \succ^* c$

1.10 Is Plurality Vote Pareto Efficient as a Social Choice Function?

We say a social choice function is Pareto if anytime there is some pair of outcomes b and c where b is unanimously better than c ($b \succ_i c$ for everyone) then c cannot be a social choice.

The social choice using plurality vote always Pareto efficient. It never chooses an outcome such that there is some other outcome that everything thinks is strictly better.

Proof. Suppose it pick an outcome b as the social and there is some other outcome a which is unanimously better. ($a \succ_i b$).

Since a is ranked above b for everone, b can't possibly get any votes and so it cannot win. This is a contradiction.

1.11 Is Veto Pareto Efficient as a Social Choice Function?

$b \succ c \succ a$

$b \succ c \succ a$

$b \succ c \succ a$

b and c are the social choices since they get the fewest vetos. Yet, b is unanimously better than c.

1.12 IIA Plurality Vote

A preference aggregation rule is IIA if for two sets of preference for which there is a pair of outcomes where every individual has the same preferences over those two outcomes in both sets, then the social preference have to be the same for that pair from both sets.

Let's do a counter-example for three people and three outcomes.

$a \succ b \succ c$

$b \succ a \succ c$

$c \succ a \succ b$

$a \sim^* b \sim^* c$

Let's change the preferences but keep the relationship between a and b for each person.

$a \succ b \succ c$

$b \succ a \succ c$

$a \succ b \succ c$

$a \succ^* b \succ^* c$

1.13 IIA Veto

$a \succ b \succ c$

$b \succ a \succ c$

$c \succ a \succ b$

$a \succ^* b \succ^* c$

$a \succ b \succ c$

$b \succ c \succ a$

$c \succ a \succ b$

$a \sim^* b \sim^* c$

1.14 Show that Borda Count is not IIA

1.15 Show that Copeland's Method is not IIA

1.16 Rule that is Transitive, Complete, IIA

a, b, c. Rule that always assigns $a \succ^* b \succ^* c$.

2 Strategic Voting

2.1 Private Information

We say preferences are **Private information** if the decision-maker/administrator does not know what they are.

When preferences are private information, when we ask for the preferences there is a chance the individuals might lie to us especially if it is in their best interest.

2.2 Decisive Social Choice

$a \succ b \succ c$

$a \succ b \succ c$

$c \succ b \succ a$

$c \succ b \succ a$

$b \succ c \succ a$

Plurality vote chooses a or c. (Two social choices).

When a social choice function can end in a tie (multiple social choices) we say it is **not decisive.**

A social choice function is decisive if there is always exactly one social choice.

Pluarlity vote with alphabetic tie-breaker is the same as plurality vote but breaks ties by alphabetic order (lowest wins).

This rule is decisive and it picks a as the winner.

2.3 Manipulation

$a \succ b \succ c$

$a \succ b \succ c$

$c \succ b \succ a$

$c \succ b \succ a$

$b \succ c \succ a \rightarrow c \succ b \succ a$

a wins in a pluarlity vote with tie-breaker. If person 5 lies and says their preferences are $c \succ b \succ a$ c wins and 5 likes this better.

This is an example of **strategic voting.**

We will say a social choice function is **non-manipulable** if there is never incentive for a person to misstate their preferences. They can never lie about their preferences and get an outcome they like better.

2.4 Gibbard-Satterthwaite

The only social choice function that is **decisive, pareto efficient**, and **non-manipulable** is a dictatorship.

This tells us we sort of have to be ok with the possibility of strategic voting.

3 Single-Peaked Preferences

Restricted Domain.