$$u_i(t) = 20\sqrt{n * t} - t$$

$$t \to 100\,(1000) = 100000$$

0.1 Public Goods Different Preferences

$$u_i(g_i, g_{-i}) = a_i\sqrt{g_i + g_{-i}} - g_i$$

For instance, $a_1 = 10$,$a_2 = 20$. Then person one cares about the park half as much as person.

Another example:

$a_1 = 10, a_2 = 20, a_3 = 60$.

0.2 Equilibrium

The best straegy for person i is to choose the g_i that maximizes their utility subject to the total chosen by everyone else.

$$\frac{\partial\left(a_i\sqrt{g_{-i} + g_i} - g_i\right)}{\partial g_i} = 0$$

$$\frac{a_i}{2\sqrt{g_{-i} + g_i}} - 1 = 0$$

$$\frac{a_i}{2\sqrt{g_{-i} + g_i}} = 1$$

$$a_i = 2\sqrt{g_{-i} + g_i}$$

$$\frac{1}{2}a_i = \sqrt{g_{-i} + g_i}$$

$$\frac{1}{4}a_i^2 = g_{-i} + g_i$$

$$g_i = \frac{1}{4}a_i^2 - g_{-i}$$

0.3 Example for Two People

$a_1 = 10$,$a_2 = 20$

$$g_1 = 25 - g_2$$

$$g_2 = 100 - g_1$$

Individually ideal total contributions are 25,100 respectively for person 1 and 2.

In a model with different preferences, in equilibrium the total contributions will be equal to the **highest** individually optimal total contributions and the only people who contribute are those who have the highest individually optimal contributions.

In this case, in equilibrium the total contributions are 100 and 2 contributes everything. $(0, 100)$.

0.3.1 Examples

$a_1 = 10, a_2 = 20, a_3 = 60$

$$g_1 = 25 - g_{-1}$$

$$g_2 = 100 - g_{-2}$$

$$g_3 = 900 - g_{-3}$$

In equilibrium, $g = 900$ and 3 contributes everything.

$$(0, 0, 900)$$

$a_1 = 10, a_2 = 20, a_3 = 60, a_4 = 60$

$$(0, 0, 600, 300)$$

$$(0, 0, 900, 0)$$

$$(0, 0, 0, 900)$$

$$(0, 0, 450, 450)$$

0.4 Utilitarian Max

What level of contributions are ideal for society? What g maximizes utilitarian welfare.

The utilitarian welfare is:

$$\frac{1}{n}\sum_{i=1}^{n}\left(a_i\sqrt{g_i+g_{-i}}-g_i\right)$$

$$\frac{1}{n}\sum_{i=1}^{n}a_i\sqrt{g_i+g_{-i}}-\frac{1}{n}\sum_{i=1}^{n}g_i$$

$$\frac{1}{n}\sum_{i=1}^{n}a_i\sqrt{g}-\frac{1}{n}\sum_{i=1}^{n}g_i$$

$$\frac{1}{n}\sum_{i=1}^{n}a_i\sqrt{g}-\frac{1}{n}g$$

$$\sqrt{g}\left(\frac{1}{n}\sum_{i=1}^{n}a_i\right)-\frac{1}{n}g$$

Call average a_i $\overline{a}$ $\overline{a}=\frac{1}{n}\sum_{i=1}^{n}a_i$

For example, when $a_1=10, a_2=20, a_3=60$, $\overline{a}=30$.

$$\overline{a}\sqrt{g}-\frac{1}{n}g$$

To maximize this, take the derivative with respect to g

$$\frac{\partial\left(\overline{a}\sqrt{g}-\frac{1}{n}g\right)}{\partial g} = \frac{\overline{a}}{2\sqrt{g}}-\frac{1}{n}$$

$$\frac{\overline{a}}{2\sqrt{g}}-\frac{1}{n}=0$$

$$g=\frac{\overline{a}^2n^2}{4}$$

This is the utilitarian ideal contributions. We can find the utilitarian ideal tax by dividing this by n.

$$t=\frac{\overline{a}^2n^2}{4n}=\frac{1}{4}\overline{a}^2n$$

In this case where $a_1=10, a_2=20, a_3=60$

$$g=2025, t=675$$

0.5 Ideal Tax for Each Individual?

$$u_i(t) = a_i\sqrt{n * t} - t$$

Ask person i "what is your favorite tax"?
The answer is whatever t maximizes u_i:

$$t = \frac{1}{4}a_i^2 n$$

$a_1 = 10, a_2 = 20, a_3 = 60$
The individually ideal taxes are: $75, 300, 2700$ respectively.
Compare this to the utilitarian ideal of 675

$$t = \frac{1}{4}\overline{a}^2 n$$

0.6 Vote

What if you didn't know what tax was optimal? What if you didn't know the preferences?

One way would be to ask for preferences and use a social choice function or preference aggregation rule.

Let's say we wanted to choose between three taxes $75, 300, 2700$.

Let's apply majority rule preference aggregation rule to these taxes.

$$\left(\begin{array}{ccc} & & \\ i = 1 : 75 & 0 & -1800 \\ i = 2 : 225 & 300 & -900 \\ i = 3 : 825 & 1500 & 2700 \end{array}\right)$$

Let's put up every tax against every other one in a pairwise vote.
75 vs 300: 300 wins getting votes from $2, 3$
2700 vs 300: 300 wins getting votes from $1, 2$
75 vs 2700: 75 wins getting votes from $1, 2$

$300 \succ^* 75 \succ^* 2700$

In fact, 300 would win a pairwise vote against **any other tax** including ones not shown here. In this sense, it is the only tax (outcome) where there is no other outcome preferred by a majority of people.

Here we call 300 the "median voter's" favorite tax. The median voter's favorite tax will always be a **condorcet winner** in a public goods model. That is the tax for which there is no other tax a majority of people prefer.

0.7 Median Mechanism

Ask each person "What is your favorite tax?"

Find the median of those numbers and make that the tax.

$75, 300, 2700$

No only does this implement the same outcome as the majority rule vote, it does it in a way that non-manipulable.