1 Mechanism Design - Public Goods

A public good costs c to build.

Each person has value v_i for the public good.

If it is built, they get utility v_i otherwise they get utility 0.

1.1 Efficiency

Utilitarian welfare if the park is built:

$$\frac{\sum_{i=1}^{n}(v_i) - c}{n}$$

If it is not built,

$$\frac{0}{n}$$

It is efficient to build if:

$$\frac{\sum_{i=1}^{n}(v_i) - c}{n} > \frac{0}{n}$$

$$\sum_{i=1}^{n}(v_i) - c > 0$$

$$\sum_{i=1}^{n} v_i > c$$

1.2 Goals?

Ultimate goal is to construct a way of learning about v_i and also implementing the efficient outcome.

Whatever set of incentives we use here:

Incentive Compatibility: In the mechanism no one has incentive to lie about their v_i.

Efficient: Given the set of valuations v_i, the mechanism should always implement the efficient decision.

A mechanism is a function that maps "claims" about preferences like v_i into a **decision** for society and (**transfer**) how much each person pays.

1.2.1 Example of a Mechanism

Everyone says their valuation v_i.

Decision is always "don't build"

No one pays anything.

While this is incentive compatible (no reason to lie) it is not efficient.

1.3 Example

$v_1 = 1000$,$v_2 = 2500, v_3 = 7500$

$c = 9000$

Is it efficient to build the park? Yes since $v_1 + v_2 + v_3 > c$. $11000 > 9000$.

1.4 Unanimity Mechanism

If everyone have a valuation more than $\frac{c}{n}$, build the park and charge each person $\frac{c}{n}$.

In the example above if $v_i \geq \frac{9000}{3} = 3000$ for everyone, then build:

$v_1 = 1000$,$v_2 = 2500, v_3 = 7500$

In this case it wouldn't be built.

This mechanism is always **incentive compatible**.

1.4.1 Incentive Compatibility

$v_1 = 1000$,$v_2 = 3500, v_3 = 7500$

No reason for 1 to lie and change the outcome to "build" because then they will owe 3000 even though their valuation is only 1000.

In fact, no one ever has at incentive to lie about their valuation regardless of what others say. **Incentive Compatibility.**

1.4.2 Efficiency of Unanimity Mechanism

If the public good is built, it is always efficient to do so under this mechanism.

To show this, note that if it is built, then $v_i \geq \frac{c}{n}$ for everyone. If we sum v_i we get $\sum_{i=1}^{n} v_i \geq \sum_{i=1}^{n} \frac{c}{n} = n\frac{c}{n} = c$.

Thus, $\sum_{i=1}^{n} v_i \geq c$.

However, there are scenarios where it should be built, but isn't.

1.5 Voting and the Median Mechanism

Ask each person their v_i.

Arrange the v_i in ascending order. If the one in the middle is at least $\frac{c}{n}$, build the park and charge $\frac{c}{n}$.

$v_1 = 1000$,$v_2 = 2500, v_3 = 7500$

$$(1000, 2500, 7500)$$

Median voter's valuation is $2500 < \frac{9000}{3} = 3000$ so don't build.

This is incentive compatible, but not efficient.

$v_1 = 1000$,$v_2 = 2500, v_3 = 7500$, $c = 9000$ it should be built but it isn't

$v_1 = 0$,$v_2 = 3500, v_3 = 4000$, $c = 9000$ it shouldn't be built but it is.

Here, inefficiency happens in both ways.

1.6 Naive Approach

Ask each person v_i. Build the park if $\sum v_i \geq c$. Charge each person $\frac{c}{n}$.

While this would be efficient if people tell the truth, not incentive compatible.

$v_1 = 1000$,$v_2 = 2500, v_3 = 7500$, $c = 9000$.

Suppose everyone tells the truth.

$$(1000, 2500, 7500)$$

Since sum is greater than 9000, the park is built and each person is charged $\frac{9000}{3} = 3000$.

Person 2 has incentive to lie. If they claim their valuation is 0, then the $v_1 + v_2 + v_3 = 8500 < 9000$ so the park won't be built and 2 likes this better.

1.7 Incentive Compatible and Efficient Mechanism

The only (class of) mechanism that does both is the VCG mechanism.

We ask each person their valuation v_i.

If $\sum_{i=1}^{n} v_i \geq c$ then build the public good.

Transfers: If person i is **pivotal** then they pay something.

$$t_i = c - v_{-i}$$

$v_{-i} = \sum_{j=1}^{n} v_j - v_i$. $v_{-1} = v_2 + v_3$

A person is pivotal if their existence changes the outcome.

If $v_{-i} < c$ and $v_{-i} + v_i \geq c$ we say i is pivotal.

$v_1 = 1000$,$v_2 = 2500, v_3 = 7500$, $c = 9000$.

Who is pivotal?

Person 1: Person 1 is not pivotal since $v_2 + v_3 = 10000 > 9000$.

Person 2: Person 2 is pivotal since $v_1 + v_3 = 8500 < c$ but $v_1 + v_2 + v_3 = 11000 > c$.

Person 3: Person 3 is pivotal since $v_1 + v_2 = 3500 < c$ but $v_1 + v_2 + v_3 = 11000 > c$.

What are they charged?

$$t_2 = c - v_{-2} = 9000 - (1000 + 7500) = 500$$

$$t_3 = c - v_{-3} = 9000 - (1000 + 2500) = 5500$$

Notice that there is a deficit. The total amount raised $t_1 + t_2 + t_3 = 6000 < 9000$. The mechanism is not **budget balanced**. There are scenarios like this, where there is a deficit. It doesn't raise enough money to build the park even when it is efficient.

There are *some* scenarios where it does raise enough money:

As an example:

$v_1 = 0, v_2 = 0, v_3 = 10000$

Only 3 is pivotal.

$$t_3 = 9000 - (0) = 9000$$

1.8 Impossibility

What we really need is Incentive Compatibility, Efficiency, Budget Balance.

No mechanism exists that meets all three goals.

We are left with a tradeoff.

Among incentive compatible mechanisms, we can't have efficiency and budget balance.

If we need budget balance we have to give up efficiency.