1 Game Theory

Cardinal Model:

$P = \{a, b\}$

$O = \{ab, a, b, n\}$

$u_a(ab) = 12, u_a(a) = 10, u_a(b) = 25, u_a(n) = 5u_b(ab) = 12, u_b(a) = 25, u_b(b) = 10, u_b(n) = 5$

1.1 What is a game?

Outcomes are determined by the choices (strategies) of the people.

Strategies here will be "clean" y "not clean" n.

Game

$P = \{a, b\}$

$s_a = \{y, n\}$

$s_b = \{y, n\}$

$u_a(y, y) = 12, u_a(y, n) = 10, u_a(n, y) = 25, u_a(n, n) = 5u_b(y, y) = 12, u_b(y, n) = 25, u_b(n, y) = 10, u_b(n, n) = 5$

We can summarize all of this into a table:

alice/bob	bob: y	bob: n
alice: y	$12, 12$	$10, 25$
alice: n	$25, 10$	$5, 5$

$2x2$ game.

1.2 Rock,Paper,Scissors

$P = \{1, 2\}$

$S_1 = \{r, p, s\}$

$S_2 = \{r, p, s\}$

Get 1 if you win, 0 if you tie, -1 if you lose.

$U_1(p, r) = 1, U_1(p, p) = 0, U_1(p, s) = -1$

$U_1(r, r) = 0, U_1(r, p) = -1, U_1(r, s) = 1$

$U_1(s, r) = -1, U_1(s, p) = 1, U_1(s, s) = 0$

Person 2's utility looks similar.

	r	p	s
r	$0,0$	$-1,1$	$1,-1$
p	$1,-1$	0,0	$-1,1$
s	$-1,1$	$1,-1$	0,0

1.3 Microwaving Fish the Game

Alice's strategies microwave y not n

Bob's strategies leave l stay s

alice/bob	bob: s	bob: l
alice: y	25,5	$25,10$
alice: n	20,20	$20,10$

1.4 Best Responses

How do people play games.

A person will always choose what is best for them conditional on the choice of the other person (people). Both people maximize their utility conditional on the choices of others.

Best Response is a strategy that has the highest utility conditional on some combination of choices for others.

For example. In the cleaning game, if bob chooses y, then it is best for alice to choose n.

Since when bob cleans, alices utility is highest when she doesn't clean, we say that for alice n is a best response to y.

The best she can do if bob doesn't clean is to clean. So we say, for alice y is a best response to n.

The best response to y from alice is for bob to play n. For bob, n is a best response to y

Formal Definition: We say s_i is the best response for person i to s_{-i} (some combination of strategies for the other's) if s_i gives person i the highest utility of any other strategy they could choose.

We can summarize the best responses into the **best response function.** $B_i\left(s_{-i}\right)$

For example, alices best response takes in Bob's strategy and spits out alice's best option in that condition:

$B_a\left(y\right) = n, B_a\left(n\right) = y$

$B_b\left(y\right) = n, B_b\left(n\right) = y$

alice/bob	bob: y	bob: n
alice: y	12,12	**10**,**25**
alice: n	**25**,**10**	5,5

$B_a(s) = y, B_a(l) = y$

$B_b(y) = l, B_b(n) = s$

alice/bob	bob: s	bob: l
alice: y	**25**,5	**25**,**10**
alice: n	20,**20**	20,10

For alice, y is a strictly dominant.

We say a combination of strategies s_1, s_2 is Nash equilibrium if the strategies are mutual best responses:

$B_1(s_2) = s_1, B_2(s_1) = s_2$

Another way to think about Nash equilibrium is to ask if the pair of strategies happened, would either have wanted to change their strategy in hindsight (hold the other's strategy fixed).

The nash equilibrium in **pure strategies** (no randomization) for cleaning example $(y, n), (n, y)$

In the microwaving example (y, l) is the Nash equilibrium.

1.5 RPS Nash

	r	p	s
r	0,0	-1,**1**	**1**,-1
p	**1**,-1	0,0	-1,**1**
s	-1,**1**	**1**,-1	0,0

There is no Pure strategy equilibrium in this game.

There is a mixed strategy equilibrium. It is where both picks each strategy with equal probability. $\frac{1}{3}$ probability of each. If this is the case, neither can do any better by choosing another strategy and so it is an equilibrium.

John Nash proved in his PhD dissertation in 1950 that any game with a finite number of players and a finite number of strategies will have some equilibrium, either pure or mixed. Because of this, we have name the equilibrium after him.