1 "Chain" Notation

$$a \succ b \succ c \succ d$$

Means:

$$a \succ b, a \succ c, a \succ d$$

$$b \succ c, b \succ d$$

$$c \succ d$$

Another one:

$$a \succ b \sim c \succ d$$

Means:

$$a \succ b, a \succ c, a \succ d$$

$$b \sim c, b \succ d$$

$$c \sim b, c \succ d$$

2 "Best"

The nice thing about complete and transitive preferences, is that they allow you to make choices.

If preferences are complete and transitive, then for every set of things, there is some "best" thing or things.

Best means: *It is as least as good as everything else.*

We denote the set of best things from a set B, $C(B)$. *The choice set.*

Let's use this preference relation:

$$a \succ b \sim c \succ d$$

What is "best" from $\{a, b, c, d\}$?

$$C(\{a, b, c, d\}) = \{a\}$$

What is "best" from $\{b, c, d\}$?

$$C(\{b, c, d\}) = \{b, c\}$$

b and c are at least as good as everything else.

2.1 Complete and Transitive

If a preference relation is complete and transitive, there is some "best" thing from any menu with a finite number of options.

As an example where this fails. Suppose we have preferences over $\{a, b, c\}$ but b, c are not comparable. There is no relation between them. This is an incomplete relation:

$$a \succsim b, a \succsim c, a \succsim a, b \succsim b, c \succsim c$$

$$C(\{b, c\}) = \emptyset$$

Nothing in this set is at least as good as everything else.

Suppose transitivity fails like in this relation:

$$a \succsim b, b \succsim c, c \succsim a, a \succsim a, b \succsim b, c \succsim c$$

This is a complete relation. So every menu of one or two things has a best thing or things. But what about three things?

$$C(\{a, b, c\}) = \emptyset$$

Again, if preferences are complete and transitive this kind of thing will never happen. There will always be at least one best thing for every menu.

$$a \succsim b, b \succsim c, a \succsim c, a \succsim a, b \succsim b, c \succsim c$$

Try to find the choice set from each of the menus below.

$$C(\{a\}), C(\{b\}), C(\{c\}), C(\{a, b\}), C(\{b, c\}), C(\{a, c\}), C(\{a, b, c\})$$

3 Public Models

Unlike intermediate micro, the outcomes in this course affect more than one person.

3.1 Private/Public Outcomes

A person deciding what to have for lunch at home is a private decision, it only affects them.

A person deciding what to have for lunch at on an airplane is a public decision, it does not only affect them.

Alice and Bob work in the same office. Alice can either microwave a fish or not.

3.2 Ordinal Models

A model has three things.

People involved. P

Outcomes. O

Preferences for each person over the outcomes. $\succsim_i$

3.2.1 Fish

People

$P = \{Alice, Bob\}$

Alternatives would be $\{a, b\}$ *or* $\{1, 2\}$

Outcomes

$O = \{Microwave\,Fish, Not\}$

Alternatives would be $\{yes, no\}$

Preferences

$\succsim_{Alice}$ (alice's prferences)

$\succsim_{Bob}$ (bob's preferences)

$$Microwave\,Fish \succ_a Not$$

$$Not \succ_b Microwave\,Fish$$

Let's write this in a shorter way.

$$P = \{a, b\}$$

$$O = \{y, n\}$$

$$y \succ_a n, n \succ_b y$$

3.2.2 Cleaning

Alice and bob share a work kitchen. Someone needs to clean it.

$$P = \{alice, bob\}$$

$$O = \{both, alice, bob, neither\} \quad \text{"who cleans?"}$$

$$bob \succ_a both \succ_a alice \succ_a neither$$

$$alice \succ_b both \succ_b bob \succ_b neither$$

3.3 Let's add David

Alice and Dave share a work kitchen, but Dave **loves** to clean and he is good at it. Also, Alice can't stand Dave and never wants to be around him. Dave is obsessed with Alice and loves to be around her.

$$P = \{alice, dave\}$$

$$O = \{both, alice, dave, neither\}$$

$$dave \succ_a alice \succ_a neither \succ_a both$$

$$both \succ_d dave \succ_d alice \succ_d neither$$

3.4 Cardinal Models

In a cardinal model, we assign utility to each outcome for each person rather than ordinal preferences.

3.4.1 Fish

$$P = \{a, b\}$$

$$O = \{y, n\}$$

$$U_a(y) = 10, U_a(n) = 9$$

$$U_b(y) = 1, U_b(n) = 10$$

3.4.2 Cleaning

$$P = \{alice, bob\}$$

$$O = \{both, alice, bob, neither\} \quad \text{"who cleans?"}$$

$$U_a(both) = 15, U_a(alice) = 10, U_a(bob) = 25, U_a(neither) = 5$$

$$U_b(both) = 15, U_b(alice) = 25, U_b(bob) = 10, U_b(neither) = 5$$

3.5 Exercises

3.5.1 Exercise 1

$$P = \{a, b, c\}$$

$$O = \{yes, no\}$$

$$yes \succ_a no$$

$$no \succ_b yes$$

$$yes \succ_c no$$

3.5.2 Exercise 2

Add Camden to the cleaning model. Camden is so bad at cleaning, he makes it worse than if no one had cleaned in the first place.

$$P = \{a, b, c\}$$

abc "all three". ab "Alice and Bob". n "none"

$$O = \{abc, ab, bc, ac, a, b, c, n\}$$

There are many plausible representation of Alice's possible preferences. Each tells a slightly different story. When you work this problem, try to justify the preferences you choose by explaining them in words.

$$b \succ_a ab \succ_a a \succ_a n \succ_a bc \succ_a abc \succ_a ac \succ_a c$$

$$b \succ_a ab \succ_a a \succ_a n \succ_a abc \succ_a bc \succ_a ac \succ_a c$$

$$b \succ_a ba \succ_a a \succ_a n \succ_a bc \sim_a c \succ_a abc \sim ac$$