# 1 Pareto Dominance

Cleaning example.

Both- $ab$, Alice- $a$, Bob- $b$, Neither- $n$

$$b \succ_a ab \succ_a a \succ_a n$$

$$a \succ_b ab \succ_b b \succ_b n$$

## 1.1 Pareto Dominates

We say an outcome $x$ **pareto dominates** outcome $y$ if $x$ *is at least as good for everyone as* $y$.

Formally:

$\forall i \in P$, $x \succsim_i y$.

## 1.2 Pareto As a Relation

If $x$ pareto dominates $y$, write: $xPy$.

Looking at our cleaning example, here are the preferences of the individuals.

$$b \succ_a ab \succ_a a \succ_a n$$

$$a \succ_b ab \succ_b b \succ_b n$$

What is the **pareto dominance** relation?

Because preferences are complete, everyone likes a certain outcome at least as much as itself.

$$aPa, bPb, abPab, nPn$$

What else can we say?

$$abPn, bPn, aPn$$

Is this a complete and transitive relation?

This isn't complete because it is missing relationships between $(a, b) . (a, ab) . (b, ab)$. Pareto dominance has nothing to say about outcomes where some people are better of and some people are worse off.

However, pareto dominance is always transitive. To see this, suppose everyone thinks $x$ is better than $y$ and everyone thinks $y$ is better than $z$, then of course everyone will think $x$ is better than $z$ as long as everyone has transitive preferences.

**Pareto dominance is always transitive, but it is sometimes incomplete.**

## 1.3 Undominated Elements

Because the pareto dominance relationship is not complete, there is **not always** something that pareto dominates all other outcomes. But sometimes there is...

$$a \succ_1 b \succ_1 c$$

$$a \succ_2 b \succ_2 c$$

$$aPb, bPc, aPc, aPa, bPb, cPc$$

In this instance, the Pareto dominance relationship **is** complete and transitive and $a$ pareto dominates everything else. In fact, here, for any subset of options, there is something that Pareto dominates everything else.

$$a \succ_1 b \succ_1 c$$

$$a \succ_2 c \succ_2 b$$

$$aPb, aPc, aPa, bPb, cPc$$

In this instance, the Pareto dominance relationship **is** not complete and transitive, but $a$ is still pareto dominates everything from the set $\{a, b, c\}$. But, from the set $\{b, c\}$ there is not something that Pareto dominates everything else.

Since Pareto dominance is not complete, it can't always tell us what to choose, but it can at least tell us what to eliminate.

## 1.4 Pareto Efficient

If $xPy$ but $y\not{P}x$, we say $x$ strictly Pareto dominates $y$.

A **Pareto Efficient** outcome is one that is not strictly Pareto dominated.

There will **always** be at least one Pareto efficient outcome.

## 1.5 Another Definition of Pareto Efficient.

Suppose $x$ was not Pareto efficient. Then there is something that strictly Pareto dominates it. There must be some $y$ such that $yPx$ but $x\not P y$.

That would mean by $yPx$: $y$ is at least as good for everyone as $x$.

And it would mean $x\not P y$: someone must like $y$ strictly better than $x$.

**Pareto Efficient:** *there is no y that is at least as good for everyone and strictly better for at least someone.*

Similarly, this says that if you make someone strictly better off, it can't be that everyone else is at least as well off. Someone must be made strictly worse off. This gives us a third definition:

**Pareto Efficient:** *can't make anyone strictly better off without making someone strictly worse off.*

## 1.6 Exercises

$A \succ_a B \succ_a C$

$B \succ_b A \succ_b C$

What are the Pareto Efficient outcomes?

$A$ and $B$ are Pareto efficient.

$A \sim_a B \succ_a C$

$B \sim_b A \succ_b C$

What are the Pareto Efficient outcomes?

$A$ and $B$ are Pareto efficient.