1 Preference Aggregation Rules

1.1 Social Preferences

This is a preference relation that is meant to represent "society's" preferences over outcomes. Another way to think about it is that it represents the preferences of the person making decisions for society.

$$b \succ_a ab \succ_a a \succ_a n$$

$$a \succ_b ab \succ_b b \succ_b n$$

One possible social preference:

$$ab \succ^* a \sim^* b \succ^* n$$

1.2 Preference Aggregation Rule

Aka. Social Welfare Function

A rule that turns the set of individual preferences into a social preference.

1.3 Strict Preferences

For the next several lectures, I will assume that every individual in our models has strict preferences.

For every person's preference relation, $\succsim_i$, there is **no pair** of distinct outcomes $x \neq y$ such that $x \sim_i y$.

Everyone's preferences are like this:

$$a \succ_i b \succ_i c \succ_i .....$$

No indifference.

1.4 Running Examples

1.4.1 Example 1

There are three people. They have these preferences:

1: $a \succ b \succ c$

2: $a \succ c \succ b$

3: $c \succ a \succ b$

1.4.2 Example 2

There are five people. They have these preferences:

1: $a \succ c \succ b$

2: $a \succ c \succ b$

3: $b \succ c \succ a$

4: $b \succ a \succ c$

5: $c \succ a \succ b$

1.5 Dictatorship

Rule: We pick a person. The social preferences are that person's preferences.

For the examples, let's assume person 1 is the dictator.

1.5.1 Example 1

There are three people. They have these preferences:

1: $a \succ b \succ c$

2: $a \succ c \succ b$

3: $c \succ a \succ b$

$$a \succ^* b \succ^* c$$

1.5.2 Example 2

There are five people. They have these preferences:

1: $a \succ c \succ b$

2: $a \succ c \succ b$

3: $b \succ c \succ a$

4: $b \succ a \succ c$

5: $c \succ a \succ b$

$$a \succ^* c \succ^* b$$

1.6 Unanimity Rule

Rule: $x \succ^* y$ if $x \succ_i y$ for everyone.

1.6.1 Example 1

There are three people. They have these preferences:

1: $a \succ b \succ c$

2: $a \succ c \succ b$

3: $c \succ a \succ b$

$a \succ^* b$

This is an incomplete social preference.

1.6.2 Example 2

There are five people. They have these preferences:

1: $a \succ c \succ b$

2: $a \succ c \succ b$

3: $b \succ c \succ a$

4: $b \succ a \succ c$

5: $c \succ a \succ b$

Unanimity gives us nothing.

1.7 Majority Rule

Aka. Pairwise Voting

Rule: $x \succ^* y$ if more than half of the people prefer x to y.

1.7.1 Example 1

There are three people. They have these preferences:

1: $a \succ b \succ c$

2: $a \succ c \succ b$

3: $c \succ a \succ b$

$$a \succ^* b, a \succ^* c, c \succ^* b$$

$$a \succ^* c \succ^* b$$

1.7.2 Example 2

There are five people. They have these preferences:

1: $a \succ c \succ b$

2: $a \succ c \succ b$

3: $b \succ c \succ a$

4: $b \succ a \succ c$

5: $c \succ a \succ b$

Try this one at home.

1.7.3 Condorcet Paradox

Marquis de Condorcet

1: $a \succ b \succ c$

2: b$\succ c \succ a$

3: $c \succ a \succ b$

$$a \succ^* b, b \succ^* c, c^* \succ a$$

Intransitive Social Preference Relation

1.8 Round-Robin

Aka. Copeland's Method

Rule: Conduct a pairwise vote for every pair. If an outcome wins a vote, add one to its score. The social preferences are ranked by score. So it x gets a higher score than y it is ranked higher.

1.8.1 Example 1

There are three people. They have these preferences:

1: $a \succ b \succ c$

2: $a \succ c \succ b$

3: $c \succ a \succ b$

a beats b. a beats c. c beats b.

$$a : 2, c : 1, b : 0$$

$$a \succ^* c \succ^* b$$

$$a :, b :, c :$$

1.8.2 Example 2

There are five people. They have these preferences:

1: $a \succ c \succ b$

2: $a \succ c \succ b$

3: $b \succ c \succ a$

4: $b \succ a \succ c$

5: $c \succ a \succ b$

Try this one at home.

1.8.3 Condorcet Example

1: $a \succ b \succ c$

2: b$\succ c \succ a$

3: $c \succ a \succ b$

a beats b,b beats c, c beats a

$$a : 1, b : 1, c : 1$$

$$a \sim^* b \sim^* c$$

1.9 Plurality Vote

Rule: The **score** of an outcome is the number of people who like that outcome best.

1.9.1 Example 1

There are three people. They have these preferences:

1: $a \succ b \succ c$

2: $a \succ c \succ b$

3: $c \succ a \succ b$

$$a : 2, b : 0, c : 1$$

$$a \succ^* c \succ^* b$$

1.9.2 Example 2

There are five people. They have these preferences:

1: $a \succ c \succ b$

2: $a \succ c \succ b$

3: $b \succ c \succ a$

4: $b \succ a \succ c$

5: $c \succ a \succ b$

$$a : 2, b : 2, c : 1$$

$$a \sim^* b \succ^* c$$

1.10 Plurality with Elimination

Aka. Instant Runoff

Elimination Method.

Rule. In the first round, we do a vote over all the outcomes. The outcome with the least votes is eliminated and ranked last. In the next round, we run a vote over the remaining outcomes and eliminate the one with the least votes again. This continues until all but one outcome is eliminated.

1.10.1 Example 1

There are three people. They have these preferences:

1: $a \succ b \succ c$

2: $a \succ c \succ b$

3: $c \succ a \succ b$

Round 1. $a : 2, c : 1, b : 0$. b is eliminated.

1: $a \succ c$

2: $a \succ c$

3: $c \succ a$

Round 2. $a : 2, c : 1$. c is eliminated.

$$a \succ^* c \succ^* b$$

1.10.2 Example 2

There are five people. They have these preferences:

1: $a \succ c \succ b$

2: $a \succ c \succ b$

3: $b \succ c \succ a$

4: $b \succ a \succ c$

5: $c \succ a \succ b$

Round $1 : a : 2, b : 2, c : 1$. c is eliminated.

1: $a \succ b$

2: $a \succ b$

3: $b \succ a$

4: $b \succ a$

5: $a \succ b$

Round 2 : $a : 3, b : 2$. b is eliminated.

$$a \succ^* b \succ^* c$$

1.11 Veto-Score

Rule: Subtract one from the score of an outcome for everyone who ranks it last.

1.11.1 Example 1

There are three people. They have these preferences:

1: $a \succ b \succ c$

2: $a \succ c \succ b$

3: $c \succ a \succ b$

$$a : 0, b : -2, c : -1$$

$$a \succ^* c \succ^* b$$

1.11.2 Example 2

There are five people. They have these preferences:

1: $a \succ c \succ b$

2: $a \succ c \succ b$

3: $b \succ c \succ a$

4: $b \succ a \succ c$

5: $c \succ a \succ b$

$$a : -1, b : -3, c : -1$$

$$a \sim^* c \succ^* b$$

1.12 Veto-Elimination

Rule. In each round the outcome that is the most people's least favorite is eliminated.

1.12.1 Example 1

There are three people. They have these preferences:

1: $a \succ b \succ c$

2: $a \succ c \succ b$

3: $c \succ a \succ b$

Round 1. b is the least favorite of 2 people. It is eliminated.

1: $a \succ c$

2: $a \succ c$

3: $c \succ a$

Round 2. c is the least favorite of 2 people. It is eliminated.

$$a \succ^* c \succ^* b$$

1.12.2 Example 2

There are five people. They have these preferences:

1: $a \succ c \succ b$

2: $a \succ c \succ b$

3: $b \succ c \succ a$

4: $b \succ a \succ c$

5: $c \succ a \succ b$

Round 1. b is three people's least favorite. It is eliminated.

1: $a \succ c$

2: $a \succ c$

3: $c \succ a$

4: $a \succ c$

5: $c \succ a$

Round 2. c is three people's least favorite. It is eliminated.

$$a \succ^* c \succ^* b$$

1.13 Borda

Rule: If there are 3 outcomes. A first ranking gives a score of 3, a second ranking gives a score of 2 and a third ranking gives a score of 1.

1.13.1 Example 1

There are three people. They have these preferences:

1: $a \succ b \succ c$

2: $a \succ c \succ b$

3: $c \succ a \succ b$

$a : 3 + 3 + 2 = 8$

$b : 2 + 1 + 1 = 4$

$c : 1 + 2 + 3 = 6$

$$a \succ^* c \succ^* b$$

1.13.2 Example 2

There are five people. They have these preferences:

1: $a \succ c \succ b$

2: $a \succ c \succ b$

3: $b \succ c \succ a$

4: $b \succ a \succ c$

5: $c \succ a \succ b$

$a : 11, b : 9, c : 10$

$$a \succ^* c \succ^* b$$