1 Preferences Continued

1.1 Indifference and Strict Preference

$\succsim$ weak preference relation. $x \succsim y$ says "x is at least as good as y"

$$X = \{a, b, c, d\}$$

We can define a new relation called "strict preference" $\succ$ that is true when a weak preference holds in only one direction.

$x \succ y$ when $x \succsim y$ but $y \not\succsim x$.

We can define a new relation called "indifference" $\sim$ that is true when a weak preference holds in both directions.

$x \sim y$ when $x \succsim y$ and $y \succsim x$.

Consider the following preference relation:

$a \succsim b, a \succsim c, a \succsim d, b \succsim c, b \succsim d, c \succsim d$

$a \succsim a, b \succsim b, c \succsim c, d \succsim d$

Here is what we can say about it with regard to strict and weak preference:

$$a \succ b, a \succ c, a \succ d, b \sim c, b \succ d, c \succ d$$

$$a \sim a, b \sim b, c \sim c, d \sim d$$

1.2 Sets Related to Preferences

These are sometimes called *induced sets.*

Using the example from above:

$$a \succ b, a \succ c, a \succ d, b \sim c, b \succ d, c \succ d$$

$$a \sim a, b \sim b, c \sim c, d \sim d$$

Indifference set: $\sim (b)$: "what are all the other things indifferent to b?" **AKA indifference curve.**

$$\sim (a) = \{a\}, \sim (b) = \{b, c\}, \sim (c) = \{b, c\}, \sim (d) = \{d\}$$

Strictly better than set: $\succ(b)$: "what are all the things strictly better than b?" **AKA Strict Upper Contour Set**

$$\succ(a) = \{\}, \succ(b) = \{a\}, \succ(c) = \{a\}, \succ(d) = \{a, b, c\}$$

(Weakly) Better than set: $\succ(b)$: "what are all the things strictly better than b?" **AKA Upper Contour Set**

$$\succsim(a) = \{a\}, \succsim(b) = \{a, b, c\}, \succsim(c) = \{a, b, c\}, \succsim(d) = \{a, b, c, d\}$$

You might be able to imagine what these are. Try them at home.

$\prec(a)$

$\precsim(a)$

2 Utility

A utility function $u()$ is a "mapping" from the objects/bundle to a number such that if one object/bundle is better than another, it gets a higher number. If this is true for every pair of things, we say the utility function u represents $\succsim$:

$$u(x) \geq u(y) \qquad x \succsim y$$

Looking again at our previous example:

$a \succsim b, a \succsim c, a \succsim d, b \succsim c, c \succsim b, b \succsim d, c \succsim d$

$a \succsim a, b \succsim b, c \succsim c, d \succsim d$

A utility representation:

$u(a) = 3, u(b) = 2, u(c) = 2, u(d) = 1$

Another:

$u(a) = 5, u(b) = 4, u(c) = 4, u(d) = 1$

Utility functions are (almost always) just **ordinal**. The magnitudes don't matter, all that matters is relative comparisons.

2.1 Cardinal Utility

Sometimes there is cardinal information in preferences that we can represent with a utility function. If four scoops of ice cream is worth \$2 to me but one scoop is worth just \$1 then we can say I like four scoops two times better than one scoop (with respect to how much money it is worth to me). Clearly there is information about magnitude here, and it is in the preference relation itself. Sometimes we can represent these preferences using a utility function that measures everything in terms of money, in that case the utility number is meaningful.

We often use the dollar-denominated quasi-linear utility function for this when it is reasonable. For example, let c be amount of ice cream and m be dollars.

$$u(c,m) = \sqrt{c} + m$$

What is $(4,0)$worth?

$$u(4,0) = \sqrt{4} + 0 = 2$$

Notice this is the same utility as $2 and no ice cream:

$$u(0,2) = 2$$

What is $(1,0)$worth?

$$u(1,0) = \sqrt{1} + 0 = 1$$

Notice this is the same utility as $1 and no ice cream:

Here, four of ice cream is actually worth two times more to mean one since it is worth the equivalence of two times more money.