1 Decisions Under Uncertainty

Outcomes: $A = \{a_1, ..., a_n\}$. *The set of objects or bundles.*
A simple gamble is a probability distribution over outcomes.
$A = \{apple, banana, carrot\}$

$$\left(\frac{1}{2} \circ apple, \frac{1}{2} \circ banana\right)$$

$$\left(\frac{1}{3} \circ apple, \frac{1}{3} \circ banana, \frac{1}{3} \circ carrot\right)$$

Degenerate gamble:

$$(apple)$$

We might say. If $apple \succ banana$ then it is reasonable to think this would be true:

$$(1 \circ apple, 0 \circ banana) \succ \left(\frac{1}{2} \circ apple, \frac{1}{2} \circ banana\right)$$

Without further assumptions, we could have:

$$\left(\frac{1}{2} \circ apple, \frac{1}{2} \circ banana\right) \succ (1 \circ apple, 0 \circ banana)$$

Set of Simple Gambles: $\mathcal{G}_s = \{(p_1 \circ a_1, p_2 \circ a_2, ..., p_n \circ a_n) \,|\, \sum p_i = 1, p_i \geq 0, (\forall i \in \{1, ..., n\}, a_i \in A)\}$.
A first order compound gamble is a probability distribution over simple gambles.

$$\left(\frac{1}{2} \circ \left(\frac{1}{2} \circ apple, \frac{1}{2} \circ banana\right), \frac{1}{2} \circ \left(\frac{1}{3} \circ apple, \frac{1}{3} \circ banana, \frac{1}{3} \circ carrot\right)\right)$$

Set of First-Order Compound Gambles:

$$\mathcal{G}_{c_1} = \left\{(p_1 \circ g_1, p_2 \circ g_2, ..., p_k \circ g_k) \,|\, \sum p_i = 1, p_i \geq 0, (\forall i \in \{1, ..., k\}, g_i \in \mathcal{G}_s)\right\}$$

Set of Second-Order Compound Gambles:

$$\mathcal{G}_{c_2} = \left\{(p_1 \circ g_1, p_2 \circ g_2, ..., p_k \circ g_k) \,|\, \sum p_i = 1, p_i \geq 0, (\forall i \in \{1, ..., k\}, g_i \in \{\mathcal{G}_{c_1}, \mathcal{G}_s\})\right\}$$

Set of All Compound Gambles:

$$\mathcal{G} \equiv \left\{(p_1 \circ g_1, p_2 \circ g_2, ..., p_k \circ g_k) \,|\, \sum p_i = 1, p_i \geq 0, \left(\forall i \in \{1, ..., k\}, g_i \in \mathcal{G}_{c_j}\, for\, some\, j \in \mathbb{N}\, or\, g_i \in \mathcal{G}_s\right)\right\}$$

A compound gamble is a probability distribution over compound gambles, where each of those compound gambles is of some finite order.

This is a second order compound gamble.

$$g = \left(\left(\frac{1}{2} \circ \left(\frac{1}{2} \circ \left(\frac{1}{2} \circ \$10, \frac{1}{2} \circ \$0 \right), \frac{1}{2} \circ \$5 \right) \right), \left(\frac{1}{2} \circ \$7 \right) \right)$$

Third order compound gamble.

$$g' = \frac{1}{3} \circ g, \frac{2}{3} \circ \$10$$

$$g = \left(\frac{1}{3} \circ \left(\left(\left(\frac{1}{2} \circ \left(\frac{1}{2} \circ \left(\frac{1}{2} \circ \$10, \frac{1}{2} \circ \$0 \right), \frac{1}{2} \circ \$5 \right) \right), \left(\frac{1}{2} \circ \$7 \right) \right) \right), \frac{2}{3} \circ \$10 \right)$$

1.1 Induced Simple Gambles and Reduction

Induced simple gamble. $A = \{\$0, \$5, \$7, \$10\}$

$$g = \frac{1}{2} \circ \left(\frac{1}{2} \circ \$10, \frac{1}{2} \circ \$0 \right), \frac{1}{2} \circ \$5$$

The compound gamble g induces this simple gamble:

$$g_s(g) = \left(\frac{1}{4} \circ \$10, \frac{1}{4} \circ \$0, \frac{1}{2} \circ \$5 \right)$$

Reduction. $\forall g \in \mathcal{G}, g \sim g_s(g)$. (Reduction of compound gambles) For the gambles above, if the consumer meets this assumption, then:

$$\left(\frac{1}{2} \circ \left(\frac{1}{2} \circ \$10, \frac{1}{2} \circ \$0 \right), \frac{1}{2} \circ \$5 \right) \sim \left(\frac{1}{4} \circ \$10, \frac{1}{4} \circ \$0, \frac{1}{2} \circ \$5 \right)$$