0.1 Induced Simple Gambles and Reduction

$g = \left(\left(\frac{1}{2} \circ \left(\frac{1}{2} \circ \left(\frac{1}{2} \circ \$10, \frac{1}{2} \circ \$0\right), \frac{1}{2} \circ \$5\right)\right), \left(\frac{1}{2} \circ \$7\right)\right)$.
The induced simple gamble is $g_s = \frac{1}{8} \circ \$10, \frac{1}{8} \circ \$0, \frac{1}{4} \circ \$5, \frac{1}{2} \circ \$7$. ***Reduction.*** $\forall g \in \mathcal{G}, g \sim g_s(g)$.

0.2 Expected Utility

We assume that consumers have a utility function over the outcomes.

$$u(x)$$

$X = \mathbb{R}_+$.
Linear utility.

$$v(x) = x$$

The expected property says that the utility of a gamble is equal to the weighted average of the utility over outcomes where the weights are the probability of that outcome in the simple gamble induced by the lottery.

Extend the utility function over outcomes v to utility functions over gambles u. **Expected Utility Property.** $\forall g \in \mathcal{G}, u(g) = E_g(v(a_i))$. $g = \left(\left(\frac{1}{2} \circ \left(\frac{1}{2} \circ \left(\frac{1}{2} \circ \$10, \frac{1}{2} \circ \$0\right), \frac{1}{2} \circ \$5\right)\right), \left(\frac{1}{2} \circ \$7\right)\right)$.
The induced simple gamble is $g_s = \frac{1}{8} \circ \$10, \frac{1}{8} \circ \$0, \frac{1}{4} \circ \$5, \frac{1}{2} \circ \$7$.

$$u(10) = 10$$

$$u(0) = 0$$

$$u(g) = u(g_s)$$

0.3 Expected Utility With Fruit

$$X = \{a, b, c\}$$

$$g = \frac{1}{2} \circ a, \frac{1}{2} \circ \left(\frac{1}{2} \circ b, \frac{1}{2} \circ c\right)$$

$$g_s = \frac{1}{2} \circ a, \frac{1}{4} \circ b, \frac{1}{4} \circ c$$

$$v(a) = 3, v(b) = 2, v(c) = 1$$

$$u(g) = E_g(v(x_i))$$

$$u(g) = \frac{1}{2}v(a) + \frac{1}{4}v(b) + \frac{1}{4}v(c)$$

$$u(g) = \frac{1}{2}3 + \frac{1}{4}2 + \frac{1}{4}1 = \frac{9}{4}$$

0.4 Risk Preferences

If outcomes are amounts of money. We can talk about risk preferences. $g = \left(\left(\frac{1}{2} \circ \left(\frac{1}{2} \circ \left(\frac{1}{2} \circ \$10, \frac{1}{2} \circ \$0\right), \frac{1}{2} \circ \$5\right)\right), \left(\frac{1}{2} \circ \$7\right)\right)$.
The induced simple gamble is $g_s = \frac{1}{8} \circ \$10, \frac{1}{8} \circ \$0, \frac{1}{4} \circ \$5, \frac{1}{2} \circ \$7$. $v(x) = x$

$$u(g) = \frac{1}{8}(10) + \frac{1}{8}(0) + \frac{1}{4}(5) + \frac{1}{2}(7) = 6$$

What is the expected outcomes of this gamble?

$$E_g(a_i) = 6$$

$$v(E_g(a_i)) = 6$$

In this case, the consumer is “Risk Neutral”

$$u(g) = E_g(u(a_i)) = v(E_g(a_i))$$

If we offered this consumer the gamble $\frac{1}{8} \circ \$10, \frac{1}{8} \circ \$0, \frac{1}{4} \circ \$5, \frac{1}{2} \circ \$7$ or we offered them the gamble \$6. They are indifferent.
$v(x) = 2x + 1$

$$g_s = \frac{1}{8} \circ \$10, \frac{1}{8} \circ \$0, \frac{1}{4} \circ \$5, \frac{1}{2} \circ \$7$$

$$\frac{1}{8}(v(10)) + \frac{1}{8}(v(0)) + \frac{1}{4}(v(5)) + \frac{1}{4}(v(7))$$

$$\frac{1}{8}(21) + \frac{1}{8}(1) + \frac{1}{4}(11) + \frac{1}{2}(15) \quad = \quad 13$$

$$u(g) = E_g(v(a_i)) = \frac{37}{4}$$

Expected outcome of this gamble is \$6.

$$v(6) = 13$$

This consumer is also risk neutral. **Log Utility.** $v(x) = ln(x+1)$ By jensen's inequality, because v is concave we will have $E_g(v(a_i)) < v(E_g(a_i))$.
Risk Aversion.

$$\frac{1}{8}(v(10)) + \frac{1}{8}(v(0)) + \frac{1}{4}(v(5)) + \frac{1}{4}(v(7))$$

$$u(g) = \frac{1}{8}(\log(10+1)) + \frac{1}{8}(\log(0+1)) + \frac{1}{4}(\log(5+1)) + \frac{1}{2}(\log(7+1.0)) \quad = \quad 1.7874$$

$$v(6) = \log(6+1) = 1.94591$$

0.5 Risk Preferences

Risk Preferences:
Risk Averse: $E_g(u(a_i)) < v(E_g(a_i))$
Risk Loving: $E_g(u(a_i)) > v(E_g(a_i))$
Risk Neutral: $E_g(u(a_i)) = v(E_g(a_i))$

Jensen's Inequality. *For a all random variables X $E_X(f(x)) \leq (\geq) f(E_X(x))$ if and only if f is concave (convex).*

0.6 Certainty Equivalent / Risk Premium

Certainty-Equivalent: $v(CE) = U(g)$

The **CE** is the most a consumer would pay for the gamble.

For the log utility example above:

$$v(x) = u(g)$$

$$v(x) = 1.7874$$

$$\log(x+1) = 1.7874$$

$$x = 4.9739$$

In this case the certainty equivelent of g is \$4.97. If we take the differnce between this and the expected outcome of the gamble, we get the **risk premium**.

$$6 - 4.9739 = 1.0261$$

The **risk premium** is the difference between the expected outcome of a gamble and the certainty equivalent. This represents how much money the consumer is willing to give up in expectation to avoid uncertainty.
$v(x) = x^2$

0.7 Expected Utility Theorem

Let $\succsim$ be the preference relation on $\mathcal{G}$:

Axiom 1. ***Completeness.*** $\succsim$ *is complete.*

Axiom 2. ***Transitivity.*** $\succsim$ *is transitive.*

Assume $a_1 \succ a_2 ... \succ a_n$.

Axiom 3. ***Monotonicity.*** For all $(\alpha \circ a_1, (1-\alpha) \circ a_n) \succsim (\beta \circ a_1, (1-\beta) \circ a_n)$ iff $\alpha \geq \beta$,

Axiom 4. ***Continuity/Archemedian.*** For all g $\exists p \in [0,1]$ such that $g \sim (p \circ a_1, (1-p) \circ a_n)$

Utility Representation. Under axioms $1-4$, we can represent $\succsim$ with a utility function.