0.1 Properties of Expenditure Function

$x_1 x_2$

$$x_1 = \frac{\sqrt{p_2}\sqrt{u}}{\sqrt{p_1}}, x_2 = \frac{\sqrt{p_1}\sqrt{u}}{\sqrt{p_2}}, \mu = \frac{\sqrt{p_1}\sqrt{p_2}}{\sqrt{u}}$$

Measuring the value of the constraint (utility) in terms of the objective (money). If relax my utility constraint by one unit, how much money can I save?

$$x_1 x_2$$

$$mu_1 = x_2$$

Marginal utility per dollar spent on x_1 at the optimum is:

$$\frac{mu_1}{p_1} = \frac{\frac{\sqrt{p_1}\sqrt{u}}{\sqrt{p_2}}}{p_1}$$

Suppose marginal utility per dollar spend on x_1 is 2. Then how much does it cost to increase u by one point using x_1? It is $\frac{1}{2}$.

$$\frac{p_1}{mu_1} = \frac{\sqrt{p_1}\sqrt{p_2}}{\sqrt{u}}$$

The cost of utility in terms of x_1.

$$\frac{p_2}{mu_2} = \frac{p_2}{\frac{\sqrt{p_2}\sqrt{u}}{\sqrt{p_1}}} = \frac{\sqrt{p_1}\sqrt{p_2}}{\sqrt{u}}$$

Both of these values are the multipler μ and this measures the cost of utility in dollars.

$$p_1 x_1 + p_2 x_2 + \mu\left(u - u\left(x_1, x_2\right)\right)$$

$$p_1 - \mu \frac{\partial\left(u\left(x_1, x_2\right)\right)}{\partial x_1} = 0$$

$$p_1 - \mu m u_1 = 0$$

$$\mu = \frac{p_1}{mu_1}$$

$$\mu = \frac{p_2}{mu_2}$$

$$\mu = \frac{1}{\lambda}$$

$$\mu = \frac{1}{2}$$

5. Concave in p.
6. Shephard's lemma. $\frac{\partial e}{\partial p_i} = x_i^h$

$$p_1 x_1 + p_2 x_2 + \mu \left(u - u \left(x_1, x_2\right)\right)$$

$$\frac{\partial \left(p_1 x_1 + p_2 x_2 + \mu \left(u - u \left(x_1, x_2\right)\right)\right)}{\partial p_1} = x_1^h$$

$$\frac{\partial e}{\partial p_1} = x_1^h$$

Utility $x_1 + x_2$ and prices are $p_1 = 1$ and $p_2 = 1$
Minimize the cost of achieving utility u.

$$x_1 + x_2 = u$$

$$\left(u, 0\right), \left(0, u\right), \left(\frac{1}{2}u, \frac{1}{2}u\right)$$

$$e\left(p_1, p_2, u\right) = u$$

$$\frac{\partial e}{\partial p_1} = 0$$

0.2 Properties of Demand

0.2.1 Slutsky Equation

$$x_i\left(p_1, p_2, e\left(p_1, p_2, u\right)\right) = x_i^h\left(p_1, p_2, u\right)$$

$$\frac{\partial x_i\left(p_1,p_2,e\left(p_1,p_2,u\right)\right)}{\partial p_j}=\frac{\partial\left(x_i^h\left(p_1,p_2,u\right)\right)}{\partial p_j}$$

$$\frac{\partial x_i}{\partial p_j}+\frac{\partial x_i}{\partial m}\frac{\partial e}{\partial p_j}=\frac{\partial x_i^h}{\partial p_j}$$

By Shephard's Lemma:

$$\frac{\partial x_i}{\partial p_j}+\frac{\partial x_i}{\partial m}x_j^h=\frac{\partial x_i^h}{\partial p_j}$$

$$\frac{\partial x_i}{\partial p_j}=\frac{\partial x_i^h}{\partial p_j}-\frac{\partial x_i}{\partial m}x_j$$

Substitution effect: $\frac{\partial x_i^h}{\partial p_j}$

Income effect: $\frac{\partial x_i}{\partial m}x_j$

0.2.2 Negative Own-Substitution Effects

Apply the slutsky equation to own price changes:

$$\frac{\partial x_i}{\partial p_i}=\frac{\partial x_i^h}{\partial p_i}+\left(-\frac{\partial x_i}{\partial m}x_i\right)$$

Total effect = substitution effect + income effect

For normal goods, the income effect is negative

For inferior goods, the income effect is positive

Substitution effect:

$$\frac{\partial x_i^h}{\partial p_i}=\frac{\partial\left(\frac{\partial e}{\partial p_i}\right)}{\partial p_i}=\frac{\partial e}{\partial p_i\partial p_i}$$

Since the expenditure function is concave in prices, $\frac{\partial e}{\partial p_i\partial p_i}\leq 0$

0.2.3 Elasticity

Income Elasticity $\eta_i=\frac{\frac{\partial x_i}{x_i}}{\frac{\partial y}{y}}=\frac{\partial x_i}{\partial y}\frac{y}{x_i}$

Price and Cross-Price Elasticity $\epsilon_{ij}=\frac{\frac{\partial x_i}{x_i}}{\frac{\partial p_j}{p_j}}=\frac{\partial x_i}{\partial p_j}\frac{p_j}{x_i}$.