1 Modified Cournot Models

1.1 Warm-up.

Suppose cost is $2q^2$ with inverse demand of $p = 100 - Q$. There are **two firms**.

$$\pi_1 (q_1, q_2) = q_1 (100 - (q_1 + q_2)) - 2q_1^2$$

$$\pi_2 (q_1, q_2) = q_2 (100 - (q_1 + q_2)) - 2q_2^2$$

Firm 1's first order condition:

$$-6q_1 - q_2 + 100 = 0$$

Best response for firm i:

$$q_i = \frac{1}{6} (100 - q_j)$$

To find equilibrium, *impose symmetry* (set $q_1 = q_2 = q$) on the best response.

$$q = \frac{1}{6} (100 - q)$$

$$q = \frac{100}{7.0} \approx 14.2857$$

If we had J firms:

$$q_i = \frac{1}{6} (100 - Q_{-i})$$

$$q = \frac{1}{6} (100 - (J - 1) q)$$

$$q \to \frac{100}{J + 5}$$

1.2 Stackleberg Model of Sequential Quantity-Setting

Suppose cost is $2q^2$ with inverse demand of $p = 100 - Q$. Firm one sets q_1. Firm two observes q_1 and sets q_2.

We work backwards to find the **subgame perfect nash equilibrium.** Their best response to q_1 is the same as before.

$$q_2 = \frac{1}{6} (100 - q_1)$$

We now write firm 1's profit function taking into account what firm 2 will do given firm's ones decision: q_1.

$$\pi(q_1) = q_1 \left(100 - \left(q_1 + \frac{1}{6}(100 - q_1)\right)\right) - 2q_1^2$$

$$\frac{\partial \left(q_1 \left(100 - \left(q_1 + \frac{1}{6}(100 - q_1)\right)\right) - 2q_1^2\right)}{\partial q_1} = \frac{1}{6}(q_1 - 100) - \frac{35q_1}{6} + 100$$

Optimal q_1 solves first order condition:

$$\frac{1}{6}(q_1 - 100) - \frac{35q_1}{6} + 100 == 0$$

$$q_1 = \frac{250}{17.0} = 14.7059$$

Firm two best responds:

$$q_2 = \frac{1}{6}\left(100 - \frac{250}{17.0}\right) = 14.2157$$

$$\pi_1(14.7059, 14.2157) = 14.7059(100 - (14.7059 + 14.2157)) - 2(14.7059)^2 = 612.745$$

$$\pi_2 = 14.2157(100 - (14.7059 + 14.2157)) - 2(14.2157)^2 = 606.257$$

Let's compare to the profit in the simultaneous game equilibrium:

$$\pi\left(\frac{100}{7}, \frac{100}{7}\right) = \left(\frac{100}{7}\right)\left(100 - \left(\frac{100}{7} + \frac{100}{7}\right)\right) - 2\left(\frac{100}{7}\right)^2 = 612.245$$

1.3 Collusion and Repeated Games

Suppose cost is $2q^2$ with inverse demand of $p = 100 - Q$. There are two firms.

$$\pi_1(q_1, q_2) = q_1(100 - (q_1 + q_2)) - 2q_1^2$$

Suppose the firms got together and decided to agree on a quantity q to maximize their profits.

They solve this problem. Maximize:

$$\pi_1(q, q) = q(100 - (q + q)) - 2q^2$$

$$100 - 8q = 0$$

$$q = \frac{100}{8} = 12.5$$

$$\pi_i \left(\frac{100}{8}, \frac{100}{8} \right) = \frac{100}{8} \left(100 - 2\frac{100}{8} \right) - 2 \left(\frac{100}{8} \right)^2$$

$$\pi = 625$$

What would either firm want to do if they truly believe the other is going to go along with the collusion? Firm 1 wants to best respond:

$$q_1 = \frac{1}{6} \left(100 - q_2 \right)$$

$$q_1 = \frac{1}{6} \left(100 - \frac{100}{8} \right)$$

$$q_1 = \frac{175}{12.0} = 14.5833$$

If this firm devates, it earns:

$$\pi \left(14.5833, 12.5 \right) = 14.5833 \left(100 - \left(14.5833 + 12.5 \right) \right) - 2 \left(14.5833 \right)^2$$

$$\pi_1 = 638.021$$

$$\pi_2 = 598.959$$