0.1 Risk Premium.

The difference between the certainty equivalent and the expected value of the gamble. Let c be the certainty equivalent for gamble g. Then risk premium r is:

$$r = E_g(a_i) - c$$

$\left(\frac{1}{2} \circ 10, \frac{1}{2} \circ 0\right)$ expected value is 5
Suppose $c = 4$.
Risk premium 1.

1 The Firm's Problem

Minimize the cost of producing output y.

$$w \cdot x = \sum w_i x_i$$

Production constraint $f(\boldsymbol{x}) = y$

$$min\, px\, s.t.\, u(x) \geq u$$

Production is inherently cardinal.

$$2f(x)$$

Is two times more productive a technology than

$$f(x)$$

We can't take monotonic transformations in the firm's problem.

1.1 The Firm's Problem: Cost Minimization.

$$(w_1 x_1 + w_2 x_2) + \mu(y - f(x_1, x_2))$$

$$\frac{\partial((w_1 x_1 + w_2 x_2) + \mu(y - f(x_1, x_2)))}{\partial x_1} = 0$$

$$w_1 = \mu \frac{\partial f}{\partial x_1}$$

$$\frac{w_1}{\frac{\partial f}{\partial x_1}} = \mu$$

$$\frac{\partial \left(\left(w_1 x_1 + w_2 x_2 \right) + \mu \left(y - f\left(x_1, x_2 \right) \right) \right)}{\partial x_2} = 0$$

$$w_2 = \mu \frac{\partial f}{\partial x_2}$$

$$\frac{w_2}{\frac{\partial f}{\partial x_2}} = \mu$$

$mp_1 = \frac{\partial f}{\partial x_1}$ is the marginal product. How output changes when I "increase x_1 by one unit".

How much does it cost me to increase output by 1 unit using input one?

Suppose $mp_1 = 1$ but $w_1 = 2$. $\frac{w_1}{mp_1}$ is 2. This is how much is costs to increase output by one unit using x_1.

Suppose $mp_1 = 2$ but $w_1 = 1$. $\frac{w_1}{mp_1}$ is $\frac{1}{2}$. This is how much is costs to increase output by one unit using x_1.

$\frac{w_i}{mp_i}$ is the cost of increasing output by one unit using input i.

At the optimum, this has to be the same for all inputs we are using.

1.2 Cost Function

5. Concave in w.

Why?

Choose (x_1, x_2)

$$min \left(w_1 x_1 + w_2 x_2 \right)$$

1.3 An Example– Cobb Douglass Production.

Suppose the production function is $f(x) = x_1^\alpha x_2^\alpha$.

$$x_1^\alpha x_2^\alpha$$

First order conditions.

$$\mu = \frac{w_1}{\frac{\partial\left(x_1^\alpha x_2^\alpha\right)}{\partial x_1}} = \frac{w_1 x_1^{1-\alpha} x_2^{-\alpha}}{\alpha}$$

$$\mu = \frac{w_2}{\frac{\partial\left(x_1^\alpha x_2^\alpha\right)}{\partial x_2}} = \frac{w_2 x_1^{-\alpha} x_2^{1-\alpha}}{\alpha}$$

Production constraint:

$$x_1^\alpha x_2^\alpha = y$$

$$\frac{w_1 x_1^{1-\alpha} x_2^{-\alpha}}{\alpha} = \frac{w_2 x_1^{-\alpha} x_2^{1-\alpha}}{\alpha}$$

$$\frac{x_2}{x_1} = \frac{w_1}{w_2}$$

$$w_2 x_2 = w_1 x_1$$

$$Solve[\{w_2 x_2 == w_1 x_1, x_1^\alpha x_2^\alpha == y\}, \{x_1, x_2\}]$$

Conditional factor demands:

$$x_1 = y^{\frac{1}{2\alpha}} \sqrt{\frac{w_2}{w_1}}, x_2 = y^{\frac{1}{2\alpha}} \sqrt{\frac{w_1}{w_2}}$$

Cost function

$$w_1 y^{\frac{1}{2\alpha}} \sqrt{\frac{w_2}{w_1}} + w_2 y^{\frac{1}{2\alpha}} \sqrt{\frac{w_1}{w_2}} = y^{\frac{1}{2\alpha}} 2\sqrt{w_1 w_2}$$

$x_1\left(1, w\right) = \sqrt{\frac{w_2}{w_1}}, x_2\left(1, w\right) = \sqrt{\frac{w_1}{w_2}}, c\left(1, w\right) = 2\sqrt{w_1 w_2}$
Notice:

$$x_1 = y^{\frac{1}{2\alpha}} x_1\left(1, w\right)$$

Notice that the original production function is homogeneous of degree 2α.
The cost function:

$$c\left(y, w\right) = y^{\frac{1}{2\alpha}} c\left(1, w\right)$$

1.4 Another Example

$f(x_1, x_2) = ln(x_1) + ln(x_2)$

$$\frac{w_1}{\frac{\partial(\log(x_1)+\log(x_2))}{\partial x_1}} = \mu$$

$$\frac{w_2}{\frac{\partial(\log(x_1)+\log(x_2))}{\partial x_2}} = \mu$$

$$\log(x_1) + \log(x_2) = y$$

$$Solve[\{\frac{w_1}{\frac{\partial(\log(x_1)+\log(x_2))}{\partial x_1}} == \mu, \frac{w_2}{\frac{\partial(\log(x_1)+\log(x_2))}{\partial x_2}} == \mu, \log(x_1)+\log(x_2) == y\}, \{x_1, x_2, \mu\}]$$

$$x_1 = \frac{\sqrt{w_2}e^{y/2}}{\sqrt{w_1}}, x_2 = \frac{\sqrt{w_1}e^{y/2}}{\sqrt{w_2}}, \mu = \sqrt{w_1}\sqrt{w_2}e^{y/2}$$

$w_1 = w_2 = 1$

$$x_1 = e^{y/2}, x_2 = e^{y/2}, \mu = e^{y/2}$$

$$\log(x_1) + \log(x_2)$$

$$mp_i = \frac{1}{x_i}$$

$$\frac{1}{\frac{1}{e^{y/2}}} = e^{\frac{y}{2}}$$

$$\mu = \sqrt{w_1}\sqrt{w_2}e^{y/2}$$

$x_1 = \frac{\sqrt{w_2}e^{1/2}}{\sqrt{w_1}}, x_2 = \frac{\sqrt{w_1}e^{1/2}}{\sqrt{w_2}}, \mu = \sqrt{w_1}\sqrt{w_2}e^{1/2}$

1.5 Homogeneous/Homothetic Production

For homogeneous production of degree α:

$$x_i\left(y,w\right)=y^{\frac{1}{\alpha}}x_i\left(1,w\right)$$

$$c\left(y,w\right)=y^{\frac{1}{\alpha}}c\left(1,w\right)$$

For homothetic production there exists some increasing function of $f\left(y\right)$ such that:

$$x\left(y,w\right)=f\left(y\right)x\left(1,w\right)$$

$$x\left(y,w\right)=f\left(y\right)c\left(1,w\right)$$

1.6 Separability

$f\left(x_1,x_2,x_3,x_4\right)=y$

$$x_1^{\alpha}x_2^{\alpha}+\log\left(x_3\right)+\log\left(x_4\right)$$

$$f_1\left(x_1,x_2\right)+f_2\left(x_3,x_4\right)$$

Suppose I want to produce y_1 using just f_1

$$c_1\left(y_1,w\right)$$

Suppose I want to produce y_2 using just f_2

$$c_2\left(y_2,w\right)$$

$y=y_1+y_2$

$$Min\left(c_1\left(y_1\right)+c_2\left(y_2\right)\right)$$

1.7 From the example.

$$c_1\left(y_1,w\right)=y_1^{\frac{1}{2\alpha}}2\sqrt{w_1w_2}$$

$$c_2\left(y_2,w\right)=2\sqrt{w_3w_4}e^{y_2/2}$$

$$y_1^{\frac{1}{2\alpha}} 2\sqrt{w_1 w_2} + 2\sqrt{w_3 w_4} e^{y_2/2}$$

$y_1 + y_2 \geq y$
$w_1 = w_2 = w_3 = w_4 = 1$

$$y_1^{\frac{1}{2\alpha}} 2 + 2e^{y_2/2} + \mu \left(y - \left(y_1 + y_2\right)\right)$$

$$\frac{\partial \left(y_1^{\frac{1}{2\alpha}} 2 + 2e^{y_2/2} + \mu \left(y - \left(y_1 + y_2\right)\right)\right)}{\partial y_1} = \frac{y_1^{\frac{1}{2\alpha}-1}}{\alpha} - \mu$$

$$\frac{\partial \left(y_1^{\frac{1}{2\alpha}} 2 + 2e^{y_2/2} + \mu \left(y - \left(y_1 + y_2\right)\right)\right)}{\partial y_2} = e^{\frac{y_2}{2}} - \mu$$

$$Solve[\frac{y_1^{\frac{1}{2\alpha}-1}}{\alpha} == e^{\frac{y-y_1}{2}}, y_1]$$

Weakly separable.
$f\left(g_1\left(x_1, x_2\right), g_2\left(x_3, x_4\right)\right)$.
Ratio of Partials.
Strongly Separable
Ratio of Partials.

1.8 A Separable Production Problem

$f\left(x_1, x_2, x_3, x_4\right) = ln\left(x_1\right) + ln\left(x_2\right) + \left(x_3^{\frac{1}{2}} x_4^{\frac{1}{2}}\right)$.

$$\left(x_3^{\frac{1}{2}} x_4^{\frac{1}{2}}\right)$$

2 Profit

2.1 Perfect Competition

Suppose a firm's production function is $f\left(x_1, x_2\right) = x_1^{\frac{1}{4}} x_2^{\frac{1}{4}}$ and is in perfect competition. Their cost function is $c\left(q, w\right) = 2\left(w_1 w_2\right)^{\frac{1}{2}} q^2$.

2.2 The Profit Function

Properties of the profit function in perfect competition:

1. Increasing in p,
2. Decreasing in w,
3. Homogeneous of degree one in p, w
4. Convex in p, w (why)
5. Hotelling: $\frac{\partial \pi}{\partial p} = q\left(p, w\right), -\frac{\partial \pi}{\partial w_i} = x_i\left(p, w\right)$

Combining 4 and 5 we can prove that, output is increasing in price (weakly), and any input is weakly decreasing in it's own wage (this is the substitution effect for production) $\frac{\partial q}{\partial p} \geq 0$, $\frac{\partial x_i}{\partial w_i} \leq 0$.

Part I

Markets feat. Cournot