$X$- Choice set, universe of choice objects, consumption set.

The set of all "bundles" or choice objects in your model.

**Common assumptions on $X$:**

**Euclidean:** $X \subseteq \mathbb{R}^n_+$.

**Non-empty:** $X \neq \emptyset$**.**

**Closed:** $X$ is closed.

Why does it need to be closed?

Any amount of ice cream scoops up to but not including 10 scoops.

$X = [0, 10)$. There is no optimal amount of ice cream for Finn here even if he likes more rather than less.

**Convex**: $X$ is convex. (Between any two points in the set, the points between those two points are in the set.)

$$x, y \in X$$

Implies for all $t \in [0, 1]$:

$$tx + (1 - t)\, y \in X$$

Suppose $x = (1, 0)$ and $y = (0, 1)$. Then bundles like:

$$\frac{1}{2}(1, 0) + \frac{1}{2}(0, 1) = \left(\frac{1}{2}, \frac{1}{2}\right) \in X$$

$$\frac{3}{4}(1, 0) + \frac{1}{4}(0, 1) = \left(\frac{3}{4}, \frac{1}{4}\right) \in X$$

...

# 1 Preference Relations

How does a consumer choose from a budget set $B$?

There are two possibilities for a foundation of "choice".

# 2 Choice Functions

A choice function $C$ is a mapping from $B$ into itself. A selection from every possible budget.

The set of all budget sets $\mathscr{B} = P(X)$ (power set- the set of all subsets).

The formal definition of a choice set: $C : \mathscr{B} \to X$ such that if $x \in C(B)$ then $x \in B$.

$X = \{a, b, c\}$

$\mathscr{B} = P(\{a, b, c\}) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$

Here is a choice function for the set $\{a, b, c\}$:

This might be choice function for someone who likes $a$ best, $b$ second best, and $c$ third best:

$C(\emptyset) = \emptyset$, $C(\{a\}) = \{a\}, C(\{b\}) = \{b\}, C(\{c\}) = \{c\}, C(\{a, b\}) = a$,
$C(\{b, c\}) = \{b\}, C(\{a, c\}) = \{a\}, C(\{a, b, c\}) = \{a\}$

Here's a choice function that is a littler weirder

$C(\emptyset) = \emptyset$, $C(\{a\}) = \emptyset, C(\{b\}) = \{b\}, C(\{c\}) = \{c\}, C(\{a, b\}) = \{a, b\}$,
$C(\{b, c\}) = \emptyset, C(\{a, c\}) = \{c\}, C(\{a, b, c\}) = \{a, b, c\}$

Here's another weird one:

$C(\emptyset) = \emptyset$, $C(\{a\}) = \{a\}, C(\{b\}) = \{b\}, C(\{c\}) = \{c\}, C(\{a, b\}) = a$,
$C(\{b, c\}) = \{b\}, C(\{a, c\}) = \{c\}, C(\{a, b, c\}) = \{a\}$

Here's another weird one (incoherent choice):

$C(\emptyset) = \emptyset$, $C(\{a\}) = \{a\}, C(\{b\}) = \{b\}, C(\{c\}) = \{c\}, C(\{a, b\}) = a$,
$C(\{b, c\}) = \{b\}, C(\{a, c\}) = \{c\}, C(\{a, b, c\}) = \{b\}$

$\#$ (cardinality function)

$\#(\{a, b, c\}) = 3$

$\#(\mathbb{Z}) = \aleph_0$ (aleph-null, the smallest infinity, "countably infinite")

$\#(\mathbb{R}) = \aleph_1$ (aleph-one, the (possibly next-largest infinity), "uncountable")

For a finite set, $X$ with $\#(X) = n$, $\#P(X) = 2^n$

$X = \{a, b, c\}$

Represent the set $\{a\}$ with boolean vector $(1, 0, 0)$ ,$\{a, b, c\}$ $(1, 1, 1)$.

## 3 Preference Relation

The mathematical tool we use to formalize the idea of preference is the "relation".

*Example:*

"At least as tall as." on the set of humans.

Not true for 'Greg','Shaq'

Is true for 'Shaq','Greg'

Formally, a relation $\succsim$ on set $X$ is a subset of the ordered pairs of $X$.

$$\succsim \subseteq X \times X$$

For instance, if $\succsim$ is the "at least as tall as relation":

$$(Shaq, Greg) \in \succsim$$

$$(Greg, Shaq) \notin \succsim$$

$$(Greg, Greg) \in \succsim$$

The opposite of the relation can be defined easily because of the set-theoretic foundation, we just take the complement $\prec = \succsim^C$. For our example above, this becomes the "strictly shorter than" relation.

$$(Shaq, Greg) \notin \prec$$

$$(Greg, Shaq) \in \prec$$

$$(Greg, Greg) \notin \prec$$

It is often more convenient to use **infix notation.** The infix statement "$x \succsim y$" is equivalent to $(x, y) \in \succsim$. Examples from our original "at least as tall as" relation:

$$Shaq \succsim Greg \,\text{("Shaq is at least as big as Greg.")}$$

$$Greg \not\succsim Shaq$$

$$Greg \succsim Greg$$