1 Preferences

1.1 Rational Preferences

$C(B)$

All of the elements of B that are at least as good as everything else.

All of the elements of B that are not strictly worse than anything else.

Complete, **Acyclic** $\succsim$ is equivalent to non-empty choice for every finite B.

This is weaker than rationality.

Rationality (complete and transitive $\succsim$) is equivelent to Coherent Choice and Finite Non-Emptyness.

$C(\{a,b\}) = \{a\}$ $C(\{a,b,c\}) = \{b\}$

Independence of Irrelevant Alternatives (Decision Theory)

A directed graph is a cyclic if and only if every vertex-induced subgraph has a vertex with in-degree zero.

1.2 Contour Sets

Sets derived from $\succsim$.

$\sim(x) = \{y | y \in X \text{ and } y \sim x\}$

$\succsim(x)$ (upper contour set, weakly preferred set)

$\succ(x)$(strict....)

$\precsim(x)$ (lower contour set, weakly less preferred set)

$\prec(x)$

1.3 Properties: Rational Preferences

The follow two properties apply to rational preferences. (Complete and Transitive).

1.3.1 Containment

Rational $\succsim$ is equivalent to:

$$x \succsim y \Leftrightarrow \succsim(x) \subseteq \succsim(y)$$

$$\Leftrightarrow \precsim(y) \subseteq \precsim(x)$$

Example: $a \succ b \succ c \succ d$

$$\succsim(b) = \{a,b\}, \succsim(c) = \{a,b,c\}$$

$$\{a, b\} \subseteq \{a, b, c\}$$

$$\precsim (b) = \{b, c, d\}, \precsim (c) = \{c, d\}$$

1.3.2 Indifference Curves Don't Cross

If two bundles are not indifferent then their indifference sets do not overlap (have an empty intersection)
If $x \succ y$ then $\sim (x) \cap \sim (y) = \emptyset$

2 Utility

A utility function is a quantitative/**cardinal** representation of preferences. It is an assignment of numbers meant to represent these preferences.
$U : X \to \mathbb{R}$
A utility represents $\succsim$ when:

$$x \succsim y \Leftrightarrow U(x) \geq U(y)$$

$a \succ b \succ c \succ d$

$$U(a) = 4, U(b) = 3, U(c) = 2, U(d) = 1$$

$$U(a) = 30000000, U(b) = 2, U(c) = 1, U(d) = -10$$

2.1 U under Finite X

For finite X, the existence of a utility function that represents X is equivalent to $\succsim$ being complete and transitive.
The $\Rightarrow$ is somewhat trivial since the completeness and transitivity of $\geq$ is imparted on $\succsim$.
Prove $\Leftarrow$. If rational $\succsim$ then exists U that represents $\succsim$.
Define $U(x) = \#(\precsim (x))$ (cardinality of the lower contour set: how many things are in it?)
$a \succ b \succ c \succ d$

$$U(a) = 4, U(b) = 3, U(c) = 2, U(d) = 1$$

Now we prove that this represents the preferences.

$$x \succsim y \Leftrightarrow U(x) \geq U(y)$$

We will do this in class on Thursday.