1 Indifference Curves - Slope

The slope of an indifference curve measures relatively how much x_2 would the consumer give up to get a little bit of x_1.

$$u(x_1, x_2) - \bar{u} = 0$$

$$f(x_1, x_2) = 0$$

The slope of an indifference curve is called the **Marginal Rate of Substitution** and is measured by the ratio of partial derivatives:

$$MRS = -\frac{\frac{\partial u}{\partial x_1}}{\frac{\partial u}{\partial x_2}}$$

2 More Preference Assumptions

Lexicographic preferences are complete and transitive yet there is no utility function for them.

2.1 Continuous $\succsim$.

Preferences are continutous if and only if the upper and lower contour sets are closed. **Continuous** $\succsim$. A relation $\succsim$ is continuous if and only if $\forall x \in X, \succsim (x)$ & $\precsim (x)$ are closed in X. ***Continuous Representation.*** Continuous U exists that represents $\succsim \Leftrightarrow \succsim$ is continuous, complete and transitive.

2.2 U is Ordinal.

Since prefrences are ordinal but we represent them with a cardinal utility function, that utility functions cardinality is meaningless.

Perfect subtitutes prefrences: finn compares bowls of ice cream by counting the number of scoops of ice cream. More is better.

$$(2,2) \succ (1,1)$$

$u(x_1, x_2) = x_1 + x_2$

$$u(2,2) = 4, u(1,1) = 2$$

This seems to imply that finn likes $(2,2)$ two times more that $(1,1)$.

$$\tilde{u}(x_1, x_2) = (x_1 + x_2)^2$$

$$\tilde{u}(x_1, x_2) = x_1 + x_2 + 10$$

$$\tilde{u}(x_1, x_2) = ln(x_1 + x_2)$$

This utility function represents the same preferences.
Since $f(u) = u^2$. $\tilde{u}(x_1, x_2) = f(u(x_1, x_2))$
We say u represenst $\succsim$ when $u(x) \geq u(y) \Leftrightarrow x \succsim y$

$$f(u(x)) \geq f(u(y)) \Leftrightarrow u(x) \geq u(y) \Leftrightarrow x \succsim y$$

Any monotonic (increasing function) of a utility function represents the same preferences are the original.

2.3 When is U Cardinal?

Finn preferences for ice cream.
$(2, 2) \sim (4, 0)$
$(1, 1) \sim (2, 0)$
$(3, 7) \sim (10, 0)$
$u(2, 2) = 4$

$u = x_1 x_2$
$(2, 3) \sim (z, z)$

$$(4, 1) \sim (2, 2)$$

$$(1, 1) \sim (1, 1)$$

$u = (x_1 x_2)^{\frac{1}{2}}$

$$u(4, 1) = 2$$

$$(4, 1) \sim (2, 2)$$

3 Other Preference Assumption

"Well behaved".

3.1 Monotonicity.

More is better.

(Monotonic) Weak Monotonicity. $x \geq y \Rightarrow x \succsim y$ and if $\forall i$, $x_i > y_i$ then $x \succ y$.

$$(2,2) \succsim (2,1)$$

$$(2,2) \succ (1,1)$$

$$(2,1) ? (1,2)$$

Monotonicity only applies (only has teeth) when comparing bundles where one has weakly more of everything.

$$x \geq y \Leftrightarrow x_i \geq y_i \forall i$$

$$(2,2) \geq (1,1)$$

$$(1,1) \geq (1,1)$$

$$(2,1) \geq (1,1)$$

$$(1,2) ? (2,1)$$

$$x > y \Leftrightarrow \forall i, x_i \geq y_i \,\&\, \exists i, x_i > y_i$$

$$x >> y \Leftrightarrow x_i > y_i \forall i$$

This definition is equivalent to:

Strict Monotonicity. $x \geq y \Rightarrow x \succsim y$ and if $x \geq y$ and there is some i such $x_i > y_i$ then $x \succ y$.

$$(2,1) \succ (1,1)$$

Strict monotonicity requires $(2,1) \succ (1,1)$. However, weak monotonicity only implies that $(2,1) \succsim (1,1)$ which allow $(2,1) \sim (1,1)$.

Local Nonsatiation. $\forall x \in X$ and $\forall \varepsilon > 0$, $\exists x^* \in B_\varepsilon(x) \cap X$ such that $x^* \succ x$. **This ensures the consumer will always consumer on the boundary of the budget set.**

3.1.1 Convexity

Convex Preferences. $\succsim$ is **convex** if $x \succsim x' \Rightarrow t(x) + (1-t)x' \succsim x'$, $t \in [0,1]$.
Strictly convex Preferences: if $x \succsim x' \Rightarrow t(x) + (1-t)x' \succ x'$, $t \in (0,1)$.

Convexity of Contours. $\succsim(x)$ is convex if and only if $\succsim$ is a convex preference relation. $\succsim(x)$ is strictly convex if and only if $\succsim$ is a strictly convex preference relation

Quasiconvexity of U***.*** If $\succsim$ is represented by U, then $\succsim$ is (strictly) convex if and only if U is (strictly) quasi-concave.

3.1.2 Homotheticity

Homotheticity. $\forall x, y \in X, \forall t \in \mathbb{R}_+ : x \succsim y \Rightarrow$ tx$\succsim ty$,. ***Parallel along rays.*** *If u is homothetic and differentiable,* $\frac{\frac{\partial u(x)}{\partial x_i}}{\frac{\partial u(x)}{\partial x_j}} = \frac{\frac{\partial u(tx)}{\partial x_i}}{\frac{\partial u(tx)}{\partial x_j}}$.