1 The Consumer Problem / Constrained Optimization

1.1 The Lagrange Method - Some Intuition

On Board

1.2 Dual Problem - Some Intuition

On Board

1.3 The Consumer Problems

Consumer's Constrained Maximization Problem
Maximize utility subject to the constraint $p_1x_1 + p_2x_2 + ... + p_nx_n \leq m$.
Consumer's Constrained Minimization Problem
Minimize $p_1x_1 + p_2x_2 + ... + p_nx_n$ subject to $u(x) \geq \bar{u}$

By strong duality, if $\bar{u}$ is the maximum utility that can be achieved with income m then m will be the minimum amount to spend to achieve utility u and the bundle that maximizes utility also minimizes cost of achieving that utility.

1.4 Example

Maximize $u = x_1x_2$

$$x_1x_2 - \lambda(p_1x_1 + p_2x_2 - m)$$

$$\frac{\partial(x_1x_2 - \lambda(p_1x_1 + p_2x_2 - m))}{\partial x_1} = 0$$

$$\frac{\partial(x_1x_2 - \lambda(p_1x_1 + p_2x_2 - m))}{\partial x_2} = 0$$

Complementary slackness

$$\lambda(p_1x_1 + p_2x_2 - m) = 0$$

Either the constraint doesn't mind and lambda is zero
Or the constraint binds and lambda is ≥ 0
Or both.
We know the constraint is going to bind.

$$p_1x_1 + p_2x_2 = m$$

We now have three conditions:

$$\frac{\partial\left(x_1 x_2 - \lambda\left(p_1 x_1 + p_2 x_2 - m\right)\right)}{\partial x_1} = 0$$

$$\frac{\partial\left(x_1 x_2 - \lambda\left(p_1 x_1 + p_2 x_2 - m\right)\right)}{\partial x_2} = 0$$

$$p_1 x_1 + p_2 x_2 = m$$

Solve the derivatives

$$x_2 = \lambda p_1$$

$$x_1 = \lambda p_2$$

$$p_1 x_1 + p_2 x_2 = m$$

Eliminate λ from the first two:

$$x_2 p_2 = x_1 p_1$$

$$x_1 = \frac{\frac{1}{2}m}{p_1}$$

$$x_2 = \frac{\frac{1}{2}m}{p_2}$$

$$\frac{\frac{1}{2}m}{p_1 p_2} = \lambda$$

Lambda will always be the amount of extra utility I get per dollar I spend on any good at the optimum.

The extra utility I get per dollar is $\frac{\frac{1}{2}m}{p_1 p_2}$

"The shadow value"

Optimal bundle:

$$x_1 = \frac{\frac{1}{2}m}{p_1}, x_2 = \frac{\frac{1}{2}m}{p_2}$$

1.5 Properties of Indirect Utility

1. Continuous.
2. Homogeneous of degree zero in prices and income.
3. Strictly increasing in y and weakly decreasing in p.
4. Quasi-convex in (p, y).
5. Roy's Identity. $-\frac{\frac{\partial V}{\partial p_i}}{\frac{\partial V}{\partial m}} = x_i^*$ (An envelope condition.)

1.6 Example - Cost Min

$x_1 x_2$

1.7 Properties of Expenditure Function

For U that is continuous and strictly increasing, the Expenditure Function e has the following properties:

1. Continuous.
2. For $p \gg 0$, strictly increasing and unbounded above in u.
3. Increasing in p.
4. Homogeneous of degree 1 in p.
5. Concave in p.
6. Shephard's lemma. *When x_i^h is single valued,* $-\frac{\partial e}{\partial p_i} = x_i^h$

1.8 Properties of Demand

1.8.1 Slutsky Equation

1.8.2 Slutsky Equation: $\frac{\partial(x_i(p,y))}{\partial p_j} = \frac{\partial\left(x_i^h(p,\bar{u})\right)}{\partial p_j} - \frac{\partial(x_i(p,y))}{\partial y} x_j^h$.

1.8.3 Negative Own-Substitution Effects

1.8.4 Elasticity