1 Relations Recap

1.1 Interpretation

We use relations in economics to describe preference:
$x \succsim y$ "bundle x is at least as good as bundle y"
"x is preferred to y"
$(x, y) \in \succsim$

1.2 Induced Relations

Indifference relation:
Even if I like x and y the same, we say $x \succsim y$. We would also say $y \succsim x$. In that case, we write $x \sim y$. "x is indifferent to y"
We write $x \sim y$ when both $x \succsim y$ and $y \succsim x$.

$$\sim = \{(x, y) \,| x, y \in X \wedge (x, y) \in \succsim \wedge (y, x) \in \succsim\}$$

$$\sim = \{(x, y) \,| x, y \in X \wedge x \succsim y \wedge y \succsim x\}$$

The indifference relation is **symmetric.** For all $x, y \in X$ if $x \sim y$ then $y \sim x$.
Strict Preference:
We write $x \succ y$ when $x \succsim y$ and $\neg (y \succsim x)$.

$$\succ = \{(x, y) \,| x, y \in X \wedge (x, y) \in \succsim \wedge \neg ((y, x) \in \succsim)\}$$

$$\succ = \{(x, y) \,| x, y \in X \wedge x \succsim y \wedge \neg (y \succsim x)\}$$

The strict preference relation is **asymmetric.** For all $x, y \in X$ if $x \succ y$ then $\neg (y \succ x)$.
These two relations decompose the preference relation.

1.3 Properties

Reflexive: $\forall x \in X, x \succsim x$
At least as tall as. Same size as.
Not Reflexive: $\exists x \in X, \neg (x \succsim x)$
Strictly taller than.
Irreflexive: $\forall x \in X, \neg (x \succsim x)$

Strictly taller than.

Symmetric: $\forall x, y \in X$ if $x \succsim y$ then $y \succsim x$

Same size as.

Not Symmetric: $\exists x, y \in X$ such that $x \succsim y$ and not $y \succsim x$

At least as tall as.

Asymmetric: $x, y \in X$ if $x \succsim y$ then $\neg(y \succsim x)$

Strictly taller than.

Complete: $\forall x, y \in X$ **either** $x \succsim y$ **or** $y \succsim x$ **(or both)**

At least as tall as.

For every pair, there is at least one true statement.

The total opposite of complete in the spirit of "irreflexivity" would be a relation with nothing in it. $\succsim = \emptyset$. This is nonsense so we don't bother. But I just did anyway.

Incomplete: $\exists x, y \in X$ **such that not** $(x \succsim y$ **or** $y \succsim x)$

$\exists x, y \in X$ **such that** $\neg(x \succsim y)$ **and** $\neg(y \succsim x)$

Strictly taller.

Completeness precludes asymmetry. Because completeness requires $x \succsim x$ which violates asymmetry (take x, y to both be x in the definition of asymmetry to find the counterexample.

Antisymmetric: (The application of asymmetry to distinct objects.)

$\forall x, y \in X$ such that $x \neq y$, if $x \succsim y$ then $\neg(y \succsim x)$

This is relation that is asymmetric for distinct objects.

Transitive: $\forall x, y, z \in X$**, if** $x \succsim y$ **and** $y \succsim z$ **then** $x \succsim z$**.**

This implies a "linearity" to the relation.

Intransitive, Not Transitive: $\exists x, y, z \in X$ **such that** $x \succsim y$**,** $y \succsim z$ **but not** $x \succsim z$**.**

Does there exist a complete relation on at least three things such that for all triplets, $x \succsim y$**,** $y \succsim z$ **but not** $x \succsim z$**.**

1.4 Weaker Transitivity Assumptions

This is weaker than transitivity.

Anti-cyclic: $\nexists(x_1, x_2, ..., x_n \in X$ **such that** $x_i \neq x_j$ **for** $i \neq j)$ **such that** $x_1 \succ x_2 \succ ... \succ x_{n-1} \succ x_n \succ x_1$**.**

"There does not exist a sequence of distinct objects."

This assumption implies there are no strict preference cycles of any length.

Write down an example of an acyclic but intransitive relation.

1.5 Rational Relation

We say a consumer is **rational, if** they have a preference relation $\succsim$ that is **complete** and **transitive.**

$X = \{a, b, c\}$

Complete and transitive.

$\{(a, a), (b, b), (c, c), (a, b), (b, c), (a, c)\}$

Complete and intransitive.

$\{(a, a), (b, b), (c, c), (a, b), (b, c), (c, a)\}$

Complete and intransitive.

$\{(a, a), (b, b), (c, c), (a, b), (b, c), (a, c), (c, a)\}$

This is a violation of transitivity. $b \succsim c, c \succsim a$ but not $b \succsim a$.