12.4. A. $q = 12$ and $p = 26$
B. $q = \frac{24}{n+1}$. $Q = \frac{n}{n+1}24$. $p = 50 - 2\left(\frac{n}{n+1}24\right)$

12.5. Consider the Cournot model presented in this chapter with inverse demand $p(Q) = \frac{100}{Q}$ and firm cost function $c(q) = 2q^2$. If a firm will enter if it can earn positive profit and exit if will earn negative profit, how many firms will enter the market in equilibrium if it costs 10 to enter?

A. In equilibrium $q = \frac{5\sqrt{J-1}}{J}$ and $Q = 5\sqrt{J-1}$ $p = \frac{100}{5\sqrt{J-1}}$. Thus a firm's profit in equilibrium with J firms is:

$$\pi(J) = \frac{5\sqrt{J-1}}{J}\left(\frac{100}{5\sqrt{J-1}}\right) - 2\left(\frac{5\sqrt{J-1}}{J}\right)^2$$

Firm will enter if profit with $J+1$ firms is above 1:

$$\pi(J+1) = \frac{5\sqrt{J}}{J+1}\left(\frac{100}{5\sqrt{J}}\right) - 2\left(\frac{5\sqrt{J}}{J+1}\right)^2 > 1$$

$$J < 49.9808$$

Thus, firms will enter until there are 50, then they will stop entering. Since each firm is earning positive profit when there are 50, no firm will have incentive to leave and so 50 is the equilibrium.

12.6. Two firms have production function $f(x_1, x_2) = \left(\frac{1}{x_1} + \frac{1}{x_2}\right)^{-1}$. Inverse demand is $p = \frac{400}{y_1+y_2}$.

A. What are the conditional factor demands for x_1 and x_2.

$$x_1 = \frac{\sqrt{w_1}y + \sqrt{w_2}y}{\sqrt{w_1}}, x_2 = \frac{\sqrt{w_1}y + \sqrt{w_2}y}{\sqrt{w_2}}$$

B. What is the cost function for the firms?

$$c(y) = \frac{\sqrt{w_1}y + \sqrt{w_2}y}{\sqrt{w_1}}w_1 + \frac{\sqrt{w_1}y + \sqrt{w_2}y}{\sqrt{w_2}}w_2 = \left(\sqrt{w_1} + \sqrt{w_2}\right)^2 y$$

C. Under what conditions would a price-taking firm choose to produce?

$$p \geq \left(\sqrt{w_1} + \sqrt{w_2}\right)^2$$

Now assume $w_1 = w_2 = 1$**.**

D. What are the firms' best response functions in a Cournot oligopoly model?

$$c(y) = 4y$$

$$\pi(y_i, y_j) = y_i \frac{400}{y_i + y_j} - 4y_i$$

$$\frac{\partial\left(y_i \frac{400}{y_i+y_j} - 4y_i\right)}{\partial y_i} = -\frac{400y_i}{(y_i + y_j)^2} + \frac{400}{y_i + y_j} - 4$$

$$y_i = 10\sqrt{y_j} - y_j$$

E. What are the Nash equilibrium of this game?

$$(0,0),(25,25)$$

12.7. Suppose there are two firms with different cost functions: $c_1(q_1) = 2q_1$, $c_2(q_2) = 3q_2$. What are the quantities of the two firms in Nash equilibrium a cournot game between them when inverse demand is $p(Q) = 100 - Q$?

$$q_1 = 32, q_2 = 34$$

13.1. Suppose there are two firms with cost functions $c(q) = 2q$ and inverse demand of $p(Q) = 100 - Q$.

A. What is the Nash equilibrium of this cournot game?

$$q_1 = q_2 = \frac{98}{3}$$

B. What is the subgame perfect nash equilibrium if firm one moves first, firm two observes q_1 and then sets q_2?

$$q_1 = 49$$

$$q_2 = \frac{49}{2}$$

13.2. Suppose two firms each earn 10 per day if they play the Cournot Nash equilibrium. If they collude, they can each earn 20 per day. But if one deviates and best-responds to this collusion, that firm will earn 40 while the other will earn only 5. Both firms have the same discount factor β. What does β need to be to sustain collusion?

$$\beta \geq \frac{2}{3}$$