Solutions to Final Exam (2019)

December 13, 2019

Part A.

1) *True.* If both goods are being consumed, she is at an interior solution. For smooth utility, the first order condition must hold $\frac{MU_1}{p_1} = \frac{MU_2}{p_2}$ these are precisely the rates of utility increase for a marginal increase in expenditure on each good. Thus, if change in utility with respect to a marginal increase in expenditure is well defined, it must be equal for all good. Some points given for noting that these marginal increases may be undefined for non-smooth utility functions such as Leontief.

2) *True.* For homothetic preferences, the ratio of consumption is constant. Thus, when consumption of one good increases, the other increases as well. Since both respond to income in the same direction, and it is not possible for both goods to be inferior, both must be normal.

3) *True.* The cost function must be concave. Thus, it is quasi-concave. Since the price vector $(2, 2)$ is a linear combination of $(1, 3)$ and $(3, 1)$, the cost at $(2, 2)$ must be at least as large as at the end points.

Part B.

harmonicas.

A) Constant.

B) Decreasing (take second derivatives).

C) No. The firm could always double output by doubling inputs (and thus doubling cost). Thus, the cost function is, at worst, linear in h.

D) These firms use inputs in a ratio equal to the square root of the ratio of prices, where Cobb Douglas firm uses them in ratio equal to the ratio of prices. Thus, the ratio of inputs will always be "less extreme" than for Cobb Douglas firms.

E) $x_i\left(w_i, w_j, h\right) = h\left(1 + \sqrt{\frac{w_j}{w_i}}\right)$.

F) $h\left[w_1\left(1+\sqrt{\frac{w_2}{w_1}}\right)+w_2\left(1+\sqrt{\frac{w_1}{w_2}}\right)\right]=h\left[w_1+2\sqrt{w_1w_2}+w_2\right]=h\left(\sqrt{w_1}+\sqrt{w_2}\right)^2$ or equivalent.

G) $\frac{\partial\left(h\left(\sqrt{w_1}+\sqrt{w_2}\right)^2\right)}{\partial w_1}=\frac{h\left(\sqrt{w_1}+\sqrt{w_2}\right)}{\sqrt{w_1}}=h\left(1+\sqrt{\frac{w_2}{w_1}}\right)=x_1\left(h,w_1,w_2\right)$

H) $\pi\left(q\right)=\left(81-q\right)q-9q=72q-q^2$. This is maximized at $q=36$

I) $\pi_i\left(q_i,q_j\right)=\left(81-q_i-q_j\right)*q_i-9q_i$. This is maximizes when: $q_i=\frac{72-q_j}{2}$. For $q_i=q_j$ this is $q=24$.

J) Since the firms has constant marginal cost, they will act as a monopolist when colluding. Thus each firm's quantity is $\frac{36}{2}=18$. Profit is 648 per firm. Without collusion, profit is: 576. Thus, colluding is worth 72 to each firm. If the total fine is larger than 144, they won't collude.

K) Since the firms sell fewer harmonicas when colluding, total surplus is diminished. However, the existance of a Z that makes both better off than under Cournot competition implies that total surplus must increase under collusion, a contradiction.

[(work)(rap)(nap)].

A) If we let $\tilde{r}=r+1$, then G. has a Cobb Douglass production function for applause in terms of $\tilde{r},s,n$. Thus, at the optimum $\tilde{r}=s$ or $r=s-1$. Since $t=s+r$, we have $\tilde{r}=s=\frac{t+1}{2}$. However, note that if $t<1$, $r<0$. In this case there is a corner solution of $r=0$ and $s=t$.

B) $a\left(t\right)=\left(\frac{t+1}{2}\right)^{\frac{2}{3}}$ if $t\geq 1$ and $a\left(t\right)=t^{\frac{1}{3}}$ if $t<1$.

C) $\frac{\partial\left(\frac{t+1}{2}\right)^{\frac{2}{3}}}{\partial t}\frac{t}{\left(\frac{t+1}{2}\right)^{\frac{2}{3}}}=\frac{2t}{3(t+1)}$ if $t\geq 1$ and $\frac{\partial t^{\frac{1}{3}}}{\partial t}\frac{t}{t^{\frac{1}{3}}}=\frac{1}{3}$ if $t<1$.

D) We can use the same trick as in part A to get: $\tilde{r}=s=n$. Since $t=s+r+n$, $\tilde{r}=s=n=\frac{t+1}{3}$. However, if $t<2$, then $r<0$. In this case we have a corner and $r=0$ while $n=s=\frac{t}{2}$.

E) $a\left(t\right)=\frac{t+1}{3}$ if $t\geq 2$ and $\left(\frac{t}{2}\right)^{\frac{2}{3}}$ if $t<2$.

F) If $n<1$ then the function is identical to part D. However, note that $n<1$ only when $t<2$ thus here if $t<2$ $r=0$ while $n=s=\frac{t}{2}$. Otherwise, $n=1$ and the logic of part A applies but with one hour of time spend on napping. Thus, $\tilde{r}=s=\frac{t}{2}$.

G) $a\left(t\right)=\left(\frac{t}{2}\right)^{\frac{2}{3}}$ if $t\geq 2$ and $a\left(t\right)=\left(\frac{t}{2}\right)^{\frac{2}{3}}$ if $t<2$

H) G. will always try to spend one more hour working than listening to rap. If sleeping will help him, and if he is capable, he will try to spend the same amount of time napping as working. If these priorities cannot be achieved due to lack of time, he will not listen to any rap, and instead spend as much time working and napping as possible.

If G. has enough time to listen to some rap music $t>2$, there is a chance he may prefer having not slept- as long as it is not monday. However, this will only be true if he has a significant amount of time to get a good long nap. In fact, he will need a total of $t > \frac{23}{4}$ (more than 5 horus and 45 minutes) to take a long enough nap since he needs to take more than a 2 hour and fifteen minute nap to feel better than having not slept! If he does not have at least 5 hours and 45 minutes, then he is always better off having slept.

One final note. According to Mann, et al. (2013) mean presentation applause length is about 6 seconds. G. will need to have 17 hours available to achieve this in the best case scenario. In preperation he will listen to 5 horus of Kanye West music. To achieve this, he can listen to Kanye's last 6 studio albums, which sum to exactly 5 hours of listening time.

Citations:

Mann, R. P., Faria, J., Sumpter, D. J., & Krause, J. (2013). The dynamics of audience applause. Journal of The Royal Society Interface, 10(85), 20130466.