1. A consumer has utility function:

$$u(x_1, x_2, x_3) = x_1^{\alpha} x_2^{1-\alpha} + ln(x_3)$$

Assume $\alpha > 0$ *and* $p_1 = p_2 = p_3 = 1$.

A) How does the money spent on x_3 depend on α? Sketch a graph.

x_3 is separable from x_1, x_2. Let k be the amount spent on x_3.

$$x_1 = \alpha(m-k)$$

$$x_2 = (1-\alpha)(m-k)$$

$$x_3 = k$$

Now solve for optimal k.

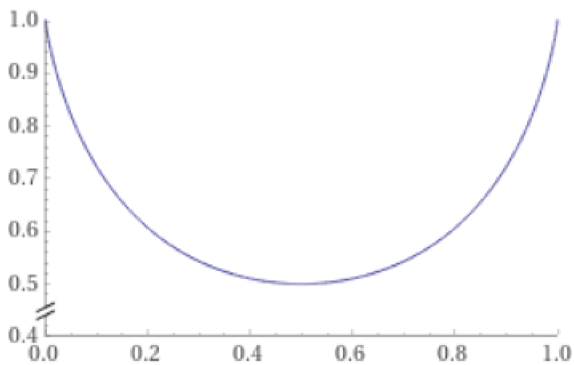
$$u(k) = (\alpha(m-k))^{\alpha}((1-\alpha)(m-k))^{(1-\alpha)} + ln(k)$$

$$= \alpha^{\alpha}(1-\alpha)^{(1-\alpha)}(m-k) + ln(k)$$

$$\frac{\partial\left(\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}(m-k) + ln(k)\right)}{\partial k} = \frac{1}{k} - (1-\alpha)^{1-\alpha}\alpha^{\alpha}$$

$$k = \frac{1}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}}$$

Plot:



B) Referencing marginal utilities, say something that makes sense of why and how money spent on x_3 depends on α in this way.

As α approaches 0 or 1 this utility function becomes quasi-linear of the form

$$x + ln\left(x_3\right)$$

In this case, the marginal utility of the remaining good is always 1. The consumer buys x_3 until its marginal utility reaches 1 ($1 spend) and then stops buying x_3. At other α levels, a dollar spent optimally on x_1 and x_2 raises utility by $(1-\alpha)^{1-\alpha}\alpha^{\alpha}$. Thus, the consumer needs to buy x_3 until the point where the marginal utility of x_3 is equal to this amount.

2. Taylor produces fancy keyboards using machine time m, aluminum a, and brass b. Machine times costs 1, aluminum costs 4, and brass costs 9. The production function for fancy keyboards is:

$$f(m, a, b) = (a + b)^{\frac{1}{4}} m^{\frac{1}{4}}$$

A) What are Taylor's conditional input demands?

Using brass here is silly. It is a perfect substitute for aluminum but costs more. Ignoring b, the cost minimizing condition is:

$$m = 4a$$

$$(a)^{\frac{1}{4}} (4a)^{\frac{1}{4}} = y$$

$$a = \frac{1}{2} y^2$$

$$m = 2y^2$$

B) What is Taylor's cost function for keyboards?

$$c(y) = 2y^2 + 2y^2 = 4y^2$$

Taylor is a monopolist for fancy keyboards, he assumes the inverse demand is $p(q) = 400 - q$:

C) how many keyboards does Taylor produce?

$$\frac{\partial \left(q (400 - q) - 4q^2\right)}{\partial q} = 400 - 10q$$

$$40 = q$$

$$p = 360$$

D) Show Taylor operates in the elastic portion of the demand curve.

Demand is:

$$q = 400 - p$$

Elasticity of demand is:

$$-\frac{\partial (400 - p)}{\partial p} \frac{p}{400 - p} \quad = \quad \frac{p}{400 - p}$$

At $p = 360$

$$\frac{360}{400 - 360} \quad = \quad 9$$

This is quite elastic.

After some market research, Taylor finds out that the demand for keyboards depends on the proportion of brass used in production. Specifically, if he uses a aluminum and b brass in production, he can sell q keyboards for $p(q) = \left(\frac{b}{a+b}\right)(400 - q)$

E) Say something about why the cost-minimization approach to profit maximization will fail in this case.

Marginal revenue depends on the chosen inputs. The cost-minimized mix of inputs may not maximize profit.

F) What is the aluminum / brass composition of the keyboards Taylor produces to maximize profit.

Profit is:

$$\pi = \left(\frac{b}{a+b}\right)\left(400 - (a+b)^{\frac{1}{4}} m^{\frac{1}{4}}\right)(a+b)^{\frac{1}{4}} m^{\frac{1}{4}} - 4a - 9b - m$$

$$= \left(400 - (a+b)^{\frac{1}{4}} m^{\frac{1}{4}}\right) \frac{b}{(a+b)^{\frac{3}{4}}} m^{\frac{1}{4}} - 4a - 9b - m$$

(Here's the hardest part of the final.) This is decreasing in a. Must be $a = 0$. This implies inverse demand will be:

$$p = (400 - q)$$

G) What is Taylor's cost function for producing keyboards of this type?

Using aluminum here is silly.

$$m = 9b$$

$$(b)^{\frac{1}{4}} (9b)^{\frac{1}{4}} = y$$

$$b = \frac{1}{3}y^2$$

$$m = 3y^2$$

$$c(y) = 3y^2 + 3y^2 = 9y^2$$

H) What is the optimal number of keyboards for him to produce?

$$\frac{\partial \left(q(400 - q) - 9q^2\right)}{\partial q} = 400 - 20q$$

$$20 = q$$

3. Firms have cost function y^2. There are J firms. Demand is $q = 100 - p$

A) The firms each assume price does not depend on their output. Find an equilibrium price p_{comp} for this market under this assumption as a function of J.

$$p_{comp} = \frac{200}{J+2}$$

$$q_{comp} = 100 - \frac{200}{J+2}$$

B) Find an expression for consumer surplus under perfect competition CS_{comp} as a function of J.

$$\frac{\left(100 - \frac{200}{J+2}\right)\left(100 - \frac{200}{J+2}\right)}{2} \quad = \quad \frac{1}{2}\left(100 - \frac{200}{J+2}\right)^2$$

C) Find a symmetric Cournot equilibrium price $p_{cournot}$ of this market as a function of J.

$$\frac{\partial\left(\left(100 - q_{-i} - q_i\right)q_i - q_i^2\right)}{\partial q_i} \quad = \quad -q_{-i} - 4q_i + 100$$

Impose symmetry:

$$q = J\frac{100}{J+3}$$

$$p_{cournot} = 100 - J\frac{100}{(J+3)}$$

$$= \frac{300}{J+3}$$

D) Show that as $J \to \infty$, p_{comp} and $p_{cournot}$ both approach 0.

Limit of both is zero...

E) What is $lim_{J\to\infty}\frac{p_{comp}}{p_{cournot}}$?

$$lim_{J\to\infty}\left(\frac{\frac{200}{J+2}}{\frac{300}{J+3}}\right) = \frac{2}{3}lim_{J\to\infty o}\left(\frac{J+3}{J+2}\right) = \frac{2}{3}$$