Econ 8100 - Final

warm-up.

A consumer has utility function $(x_1^\alpha + x_2^\alpha)^\beta$. Income is m and prices are p_1, p_2.

A. Set up the Lagrangian that turns the consumer's constrained utility maximization problem into an unconstrained maximization. Use λ for the Lagrange multiplier on the budget constraint.

B. Write down the first order conditions for this unconstrained maximization.

C. Suppose $\alpha = 2, \beta = \frac{1}{2}$. How does this consumer's utility change when one of the goods is increases marginally from a point where $x_1 = x_2 = x$?

D. Suppose $\alpha = 2, \beta = \frac{1}{2}, p_1 = p_2$. Solve for λ from part **B.**

E. Interpret the number you found in **D** using your answer to **C.**

cournot.

J firms compete in Cournot oligopoly. Each has cost function $c(y) = y + a$. Consumer demand is $q(p) = \frac{b}{p}$.

A. What are the firm's profit functions as a function of q_i and q_{-i}.

B. What is firm i's best response to quantity q_{-i} of the other firms?

C. Find the symmetric Nash equilibrium.

D. Show that as J increases, the equilibrium price approaches the firm's marginal cost from above.

E. In terms of a and b, what is the number of firms J such that profit of each firm is zero?

technologies.

A firm has access to two technologies: $f(x_1, x_2) = 2x_1^{\frac{1}{8}}x_2^{\frac{1}{8}}$ and $g(x_1, x_2) = x_1^{\frac{1}{4}}x_2^{\frac{1}{4}}$. Suppose $w_1 = w_2 = 1$.

A. Show the cost function for f is of the form $c(1)\, y^{\alpha}$ where α is the reciprocal of the degree of homogeneity of f.

B. If the firm can freely choose between these technologies, what is it's cost function?

C. What quantity does a price-taking firm produce if the price is $p = 32$.

D. Is the firms cost function from part **B** concave in y? Is it quasi-concave in y? Justify your answer.

E. Prove that the profit function is quasi-concave in x_1, x_2 for price-taking firms with the production function g.