Econ 8100 - Final Exam

December 12, 2022

1. For the choice set $X = \{(2,0), (0,2), (4,0), (0,4), (1,1), (2,2)\}$, label $a = (2,0), b = (0,2), c = (4,0), d = (0,4), e = (1,1), f = (2,2)$. Assume preferences are complete, transitive, and anti-symmetric ($x \sim y$ if and only if $x = y$) and strictly monotonic. For each of the following, provide an example as a rank ordering, or show that it is impossible. For instance writing "abcdef" as an example is acceptable.

A) $\succsim$ is homothetic and convex.

B) $\succsim$ is convex but not homothetic.

C) $\succsim$ is homothetic but not convex.

D) $\succsim$ is not homothetic and not convex.

2. A firm has production function $f(x_1, x_2, x_3) = (x_1 x_2)^{\frac{1}{3}} + x_3$. $w_1 = w_2 = 1$. $w_3 = 6$.

A) Show this function is concave.

B) What are the cost minimizing x_1, x_2, x_3?

3. A monopoly has cost function $c(q) = q^2$+c. Market demand is $q = 120 - p$. Suppose the monopolist can choose to split itself in two. If it does, both firms will have cost function $c(q) = q^2 + c$. By law, the resulting firms will have to compete in Cournot oligopoly, but *the monopolist can capture the profit of both of the resulting firms.*

A) What condition on c ensures the monopolist does not have incentive to split itself in two?

Suppose there is a potential entrant into this market with the same cost function. The monopolist first chooses whether to split. Then the potential entrant decides whether to enter. It will enter as long as the profit it earns by entering is greater than 0. *Finally, any firms in the market compete in Cournot oligopoly.*

B) What condition on c ensures the monopolist will split itself in two?

4. A Norman window has a rectangle base and is topped with a semi-circle. It looks like . Suppose you are building such a window with perimeter p.

A) Write down a Lagrangian for this constrained optimization problem.

B) Suppose at the optimum, the window is 10 units wide, what is the marginal change in achievable area as allowable perimeter increases?