Econ 8100 - Final Exam

December 12, 2023

1. For the choice set $X = \{(2,0), (0,2), (4,0), (0,4), (1,1), (2,2)\}$, label $a = (2,0), b = (0,2), c = (4,0), d = (0,4), e = (1,1), f = (2,2)$. Assume preferences are complete, transitive, and anti-symmetric ($x \sim y$ if and only if $x = y$) and strictly monotonic. For each of the following, provide an example as a rank ordering, or show that it is impossible. For instance writing "abcdef" as an example is acceptable.

A) $\succsim$ is homothetic and convex.

$cfdaeb$

B) $\succsim$ is convex but not homothetic.

$cfdbea$

C) $\succsim$ is homothetic but not convex.

$cdfabe$

D) $\succsim$ is not homothetic and not convex.

$cdfbae$

2. A firm has production function $f(x_1, x_2, x_3) = (x_1 x_2)^{\frac{1}{3}} + x_3$. $w_1 = w_2 = 1$. $w_3 = 6$.

A) Show this function is concave.

$(x_1 x_2)$ is a quasi-concave function since it is a monotonic transformation of $ln(x_1) + ln(x_2)$. $(x_1 x_2)^{\frac{1}{3}}$ is a a monotonic transformation of that and is also quasi-concave, but it is also homothetic of degree $\frac{2}{3}$ and thus concave. x_3 is concave since it is linear. The whole function is thus a sum of concave functions.

B) What are the cost minimizing x_1, x_2, x_3?

Use separability. Produce $\tilde{y}$ using x_1 and x_2. Cost is $2\tilde{y}^{\frac{3}{2}}$.

$c(y, \tilde{y}) = 2\tilde{y}^{\frac{3}{2}} + 6(y - \tilde{y})$.

$\frac{\partial\left(2z^{\frac{3}{2}}+6(y-z)\right)}{\partial z} = 3\sqrt{z} - 6$

Optimal $\tilde{y} = 4$. Thus, $x_1 = x_2 = 4^{\frac{3}{2}}, x_3 = y - 4$. If $y < 4$ then $x_1 = x_2 = y^{\frac{3}{2}}$

3. A monopoly has cost function $c(q) = q^2$+c. Market demand is $q = 120 - p$. Suppose the monopolist can choose to split itself in two. If it does, both firms will have cost function $c(q) = q^2 + c$. By law, the resulting firms will have to compete in Cournot oligopoly, but *the monopolist can capture the profit of both of the resulting firms.*

A) What condition on c ensures the monopolist does not have incentive to split itself in two?

Pays to solve this generically. The Nash equilibrium with n firms is:

$q = \frac{120}{n+3}$

Profit is:

$\pi(n) = \left(120 - n\left(\frac{120}{n+3}\right)\right)\frac{120}{n+3} - \frac{120^2}{(n+3)^2} - c$

Here are the profit of firms if there are respectively $n = 1, 2, 3$:

$1800 - c, 1152 - c, 800 - c$

When is $1800 - c > 2(1152 - c)$

$c > 504$

Suppose there is a potential entrant into this market with the same cost function. The monopolist first chooses whether to split. Then the potential entrant decides whether to enter. It will enter as long as the profit it earns by entering is greater than 0*. Finally, any firms in the market compete in Cournot oligopoly.*

B) What condition on c ensures the monopolist will split itself in two?

The monopolist chooses between splitting and not. If $c < 800$, it will enter no matter what. In that case, the monopolist chooses between getting $2(800 - c)$ and getting $1152 - c$. Will split if: $c < 448$

Suppose $1152 > c > 800$ then the entrant will enter only if the monoplist doesn't split. Chooses between $1152 - c$ and $2(1152 - c)$. Monopolist will always split.

Suppose $c > 1152$ then the entrant will never enter. We have already solved this in previous part. Monopolist always splits.

4. A Norman window has a rectangle base and is topped with a semi-circle. It looks like (See Exam). Suppose you are building such a window with perimeter p and the goal is to maximize the area of the window.

A) Write down a Lagrangian for this constrained optimization problem.

$$w * h + \frac{1}{2}\pi\left(\frac{w}{2}\right)^2 - \lambda\left(w + 2h + \frac{1}{2}\pi w - p\right)$$

B) Suppose at the optimum, the window is 10 units wide, what is the marginal change in achievable area as allowable perimeter increases?

Notice that $\lambda = \frac{w}{2}$ from the FOC on h. The answer is 5.