1. A) $x_2 = \left(\frac{1}{y} - \frac{1}{x_1}\right)^{-1}$

   B) The second derivative with respect to x_1 is negative when: $\frac{2y^2}{(x_1-y)^3} \geq 0$. This is true if $x_1 > y$. Plugging in for y, $x_1 > \left(\frac{1}{x_1} + \frac{1}{x_2}\right)^{-1}$. Notice that $\left(\frac{1}{x_1} + \frac{1}{x_2}\right)^{-1}$ is strictly increasing in x_2. Taking $x_2 \to \infty$, we get $\left(\frac{1}{x_1} + \frac{1}{x_2}\right)^{-1} < x_1$ which proves the result.

   C) Yes, convexity of the isoquants implies convexity of the upper-contour sets which implies the function is quasi-concave.

   D) Yes, of degree 1.

   E) Yes, quasi-concave functions of degree $(0, 1]$ are concave.

2. A) $-\frac{1}{2}$. A rapper who is 1% more talented than another can spend $\frac{1}{2}$% less time in the studio and produce the same amount of hits.

   B) Plug these numbers into the isoquant through $y = 10$.

   Spending no time with Kanye:

$$40\left(\frac{1}{\frac{1}{3}} + \frac{1}{x_2}\right)^{-1} = 10$$

$$x_2 = 1$$

   Total time spent is $0 + 1 = 1$

   Spending $\frac{1}{3}$ of lifetime with Kanye:

$$40\left(\frac{1}{\frac{1}{3} + \frac{1}{4}\frac{1}{3}} + \frac{1}{x_2}\right)^{-1} = 10$$

$$x_2 = \frac{5}{8}$$

   Total time spent is $\frac{1}{3} + \frac{5}{8} = \frac{23}{24}$

   It is better to spend $\frac{1}{3}$ of lifetime with Kanye.

   C) Let k be time spend with Kanye. Studio time needed to produce 10 hits is given by:

$$40\left(\frac{1}{\frac{1}{3} + \frac{1}{4}k} + \frac{1}{x_2}\right)^{-1} = 10$$

$$\left(\frac{1}{\frac{1}{3} + \frac{1}{4}k} + \frac{1}{x_2}\right)^{-1} = \frac{1}{4}$$

$$\frac{1}{\frac{1}{3} + \frac{1}{4}k} + \frac{1}{x_2} = 4$$

$$\frac{1}{x_2} = 4 - \frac{1}{\frac{1}{3} + \frac{1}{4}k}$$

$$\frac{1}{x_2} = \frac{\frac{4}{3} + k}{\frac{1}{3} + \frac{1}{4}k} - \frac{1}{\frac{1}{3} + \frac{1}{4}k}$$

$$\frac{1}{x_2} = \frac{\frac{4}{3} + k - 1}{\frac{1}{3} + \frac{1}{4}k}$$

$$x_2 = \frac{\frac{1}{3} + \frac{1}{4}k}{\frac{4}{3} + k - 1}$$

$$x_2 = \frac{\frac{1}{3} + \frac{1}{4}k}{\frac{1}{3} + k}$$

Total time spent is given by:

$$k + \frac{\frac{1}{3} + \frac{1}{4}k}{\frac{1}{3} + k}$$

$$\frac{k^2 + \frac{7}{12}k + \frac{1}{3}}{\frac{1}{3} + k}$$

To maximize leisure time, minimize this total time spent:

$$min_k \left(\frac{k^2 + \frac{7}{12}k + \frac{1}{3}}{\frac{1}{3} + k} \right)$$

The first order condition is (sorry about the quadratic):

$$\left(2k + \frac{7}{12}\right)\left(\frac{1}{3} + k\right) = \left(k^2 + \frac{7}{12}k + \frac{1}{3}\right)$$

$$k^2 + \frac{2}{3}k - \frac{5}{36} = 0$$

This gives the optimal time to spend with Kanye:

$$k = \frac{1}{6}$$

D) The minimization problem for a rapper with initial talent t is:

$$min_k \left(\frac{t + \frac{1}{4}k}{4t + k - 1} + k \right)$$

The first order condition is:

$$\frac{1}{4\left(k+4t-1\right)}=\frac{\frac{k}{4}+t}{\left(k+4t-1\right)^2}-1$$

Set $k=0$

$$\frac{1}{4\left(4t-1\right)}=\frac{t}{\left(4t-1\right)^2}-1$$

$$\frac{1}{4}=\frac{t}{\left(4t-1\right)}-\left(4t-1\right)$$

$$\frac{1}{4}+\left(4t-1\right)=\frac{t}{\left(4t-1\right)}$$

$$\left(4t-\frac{3}{4}\right)\left(4t-1\right)-t=0$$

$$16t^2-8t+\frac{3}{4}=0$$

$$t=\frac{1}{8}\ or\ \frac{3}{8}$$

However, a rapper with talent $\frac{1}{8}$ who spends no time with Kanye cannot produce 10 albums (except with a negative amount of studio time). Thus, of these two solutions, $\frac{3}{8}$ is the only valid one.

3. A) Lottery C is the simple lottery equivalent of a 50% probability of lottery A and 50% probability of lottery B. Lottery B can be written as a 0% of Lottery A and 100% probability of Lottery B. In comparing Lottery B and C, C puts more weight on the preferred lottery A. Thus, a consumer who's preference obey the reduction of compound lotteries and monotonicity will prefer B to C if they prefer A to C. The preferences of a consumer who is an expected utility maximizer obey both of these properties.

   B) Risk averse: the utility function is concave in wealth.

   C) The definition of the certainty equivalent is: $u(CE)=E(u(g))$.

$$\frac{1}{2}CE^{\frac{1}{2}}=\frac{1}{2}\left(\frac{1}{2}(w_0+10)^{\frac{1}{2}}\right)+\frac{1}{2}\left(\frac{1}{2}(w_0)^{\frac{1}{2}}\right)$$

$$CE=\left(\frac{1}{2}(w_0+10)^{\frac{1}{2}}+\frac{1}{2}(w_0)^{\frac{1}{2}}\right)^2$$

   D) There's no right answer.