Econ 8100 Midterm

October 20, 2022

1. A consumer has preferences over cups of coffee (t, m). t is temperature (in Celsius) $t \in [0, 100]$. For reference $t = 0$ is coffee ice cubes and $x = 100$ is boiling coffee. m is proportion of milk $m \in [0, 1]$. For reference $m = 0$ is black coffee and $m = 1$ is a cup of milk. Assume preferences are complete and transitive.

A. Describe a reasonable and intuitive scenario where preferences over (t, m) are not monotonic.

Likes iced coffee.

B. Describe a reasonable and intuitive scenario where preferences over (t, m) are not convex.

Prefers iced and hot coffee to room-temperature coffee.

C. Describe a reasonable and intuitive scenario where preferences over (t, m) do not meet the following assumption: $(t, m) \succsim (t, m') \Leftrightarrow (t', m) \succsim (t', m')$

Likes hot black coffee but milky iced coffee.

2. Suppose the choice set $X = \{(2,2), (3,1), (1,3), (2,0), (0,2), (1,1)\}$. Assume that $\succsim$ is complete, transitive, and anti-symmetric ($x \sim y$ if and only if $x = y$), monotonic, convex, and independent ($x \succsim y \Leftrightarrow x + a \succsim y + a$).

A. What restrictions does monotonicity put on $\succsim$?

$(2,2) \succ (1,1), (3,1) \succ (2,0), (1,3) \succ (0,2)$ (strictness comes from anti-symmetry)

B. Suppose you observe a consumer choose $(2,0)$ from the set $\{(2,0), (1,1), (0,2)\}$ and choose $(1,3)$ from the set $\{(2,0), (1,3)\}$. What is their rank ordering of the set X?

By anti-symmetry the choice of $(2,0)$ *implies* $(2,0) \succ (1,1)$ *and* $(2,0) \succ (0,2)$*. By convexity and anti-symmetry* $(1,1) \succ (0,2)$*. Thus we have the ranking* $(2,0), (1,1), (0,2)$ *over these three bundles.*

By independence we have the ranking $(3,1), (2,2), (1,3)$ *over these three bundles.*

By the choice of $(1,3)$ *over* $(2,3)$ *we have the full ranking:*

$$(3,1)\,,(2,2)\,,(1,3)\,,(2,0)\,,(1,1)\,,(0,2)$$

3. A consumer has utility function $u(x_1, x_2, x_3) = x_1 x_2 + x_3$ and budget $x_1 + x_2 + x_3 \leq m$.

A. Suppose the consumer decides to spend some part of their budget $\tilde{m}$ on x_1 and x_2 such that $x_1 + x_2 = \tilde{m}$. What is the optimal amount of x_1, x_2 to buy?

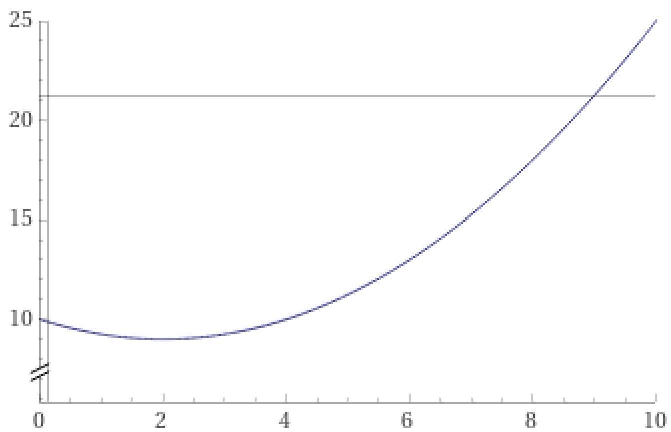
$x_1 = \frac{1}{2}\tilde{m}, x_2 = \frac{1}{2}\tilde{m}$

B. Write the consumer's achievable utility as a function of $\tilde{m}$ and m refer to this as $V(m, \tilde{m})$.

$V(m, \tilde{m}) = \left(\frac{1}{2}\tilde{m}\right)^2 + (m - \tilde{m})$

C. Sketch $V(10, \tilde{m})$

$V(10, \tilde{m}) = \left(\frac{1}{2}\tilde{m}\right)^2 + (10 - \tilde{m})$



D. Is $V(m, \tilde{m})$ convex or concave in $\tilde{m}$?

Convex.

E. What is the optimal $\tilde{m}$ as a function of m?

$\tilde{m} = m \text{ } if \text{ } m > 4$

$\tilde{m} = 0 \text{ } if \text{ } m \leq 4$

F. What is the consumer's optimal x_1, x_2, x_3 as a function of m?

$x_1 = \frac{1}{2}m, x_2 = \frac{1}{2}m, x_3 = 0 \text{ } if \text{ } m > 4$

$x_1 = 0, x_2 = 0, x_3 = m \text{ } if \text{ } m \leq 4$

G. What is this consumer's marginal utility of money as a function of m?

$\frac{m}{2} \text{ } if \text{ } m > 4$

$1 \text{ } if \text{ } m \leq 4$