1. *Firms compete in a market by setting quantity. Market demand is* $Q_d = 40 - 2p$. *They each have constant marginal costs of* 5. *Fixed costs are* 0.

a) Find the quantity for each firm in a symmetric Cournot equilibrium with J firms in this market.

b) Show that the profit per firm in this equilibrium is is $\frac{450}{(J+1)^2}$.

Suppose now a single firm could commit to quantity $\tilde{q}$ *and announce this publicly to the other* $(J-1)$ *firms who would them compete in a symmetric Cournot oligopoly.*

c) What would the $J - 1$ firms that do not commit set as their quantity as a function of $\tilde{q}$ and J?

d) When $J = 3$ what does the committing firm set as $\tilde{q}$?

2. *A individual consumes bundles in* $\mathbb{R}^2_+$ *and has a preference relation* $\succsim$ *on these bundles that can be represented by utility function* $U(x_1, x_2)$.

a) What does it mean for a utility function to be homogeneous of degree 1?

b) What does it mean for a consumer's preference to be homothetic.

c) Prove that a consumer with preferences that can be represented by a homogeneous utility function has homothetic preferences.

d) Prove when prices are fixed a consumer with homothetic preferences chooses the same ratio of the two goods regardless of income.

e) Define the *expenditure function*.

f) Prove that a consumer with a utility function that is homoegenous of degree 1 has an expenditure function that is homogeneous of degree 1 in utility.

g) Suppose it costs $10 to achieve a utility of 1 at prices $p_1 = 1$ and $p_2 = 3$. It also costs $10 to achieve a utility of 1 at prices $p_1 = 3$ and $p_2 = 1$. What can you say about the cost of achieving a utility of 1 at prices $p_1 = 2$ and $p_2 = 2$. How do you know?

3. *Haagsma (2012) discusses a family of utility functions that display Giffen behavior.* $U(x_1, x_2)$ *below is a member of that family.*

$$U(x_1, x_2) = 2ln(x_1 - 6) - 8ln(12 - x_2)$$

The consumer is constrained by income m and $x_1 > 6$, $0 \leq x_2 < 12$.

> *"Slutsky provided the by now familiar argument that the assumption of diminishing marginal utility is not necessary for downward sloping convex indifference curves. In particular, he found that in the case of additive utility an appropriate indifference map may also be obtained if one—but only one—good has increasing marginal utility."*

a) Show that one- but only one- of these goods has increasing marginal utility.

b) For the utility function $\tilde{U}$ below, write the equation for an indifference curve through bundles with utility $\bar{u}$. This should be written as $x_2 = f(x_1, \bar{u})$.

$$\tilde{U}(x_1, x_2) = \frac{(x_1 - 6)^2}{(12 - x_2)^8}$$

c) How do we know that the indifference curves of $\tilde{U}$ are the same as the indifference curves of U?

d) Prove that these indifference curves are downward sloping and convex.

e) Suppose $p_1 > 2$,$p_2 = 1$, and $m = 24$, find a condition on p_1 where x_1 is Giffen.

f) Explain why, when $p_1 < 2$, there is no "optimal bundle".