1. A firm has production function $f(x_1, x_2) = \left(\frac{1}{x_1} + \frac{1}{x_2}\right)^{-\frac{1}{2}}$.

A) Show this function is increasing in x_i for $x_1 > 0, x_2 > 0$.

B) What is the degree of homogeneity for this function?

For the rest of the problem, suppose the input prices for x_1 and x_2 are $w_1 = 4$ and $w_2 = 1$.

C) What are the firm's conditional factor demands?

D) What is the firm's cost function.

E) Say something interesting about the relationship between your answer to part B and your answer to part D.

2. Suppose firms have cost function $c(y) = 9y^2$ and market demand is $100 - p$.

A) How much output does a monopolist produce? What does it charge?

B) How much does *each firm produce* in Cournot oligopoly with N firms?

C) The government really wants to have 81 firms in this market for some reason. They decide to charge a license fee F to any firm operating in the market. What is an amount F the government can set so that 81 firms enter the market?

3. A consumer has utility function $u = ln\left(x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}\right) + x_3$ and income m. Prices are p_1, p_2, p_3. Assume $p_3 = 1$ throughout the problem.

A) Show that this function is concave. Do this however you'd like.

B) Set up the Lagrangian from which the unconstrained maximum gives the consumer's constrained utility maximum. Use λ for the multiplier.

C) Using the derivatives of the Lagrangian, argue that: If the consumer is optimizing and consumes some of each good, the consumer's marginal utility for money is 1.

D) What are this consumer's demands for x_1, x_2 and x_3? Account for any corner solutions.

4. $\succsim$ is a binary relation that is complete and transitive on some finite set X. Let $\succ$ be defined as follows: $x \succ y \Leftrightarrow x \succsim y$ and not $y \succsim x$. **Show that $x \succ y$ and $y \succsim z$ implies $x \succ z$.**