1. A firm has production function $f(x_1, x_2) = \left(\frac{1}{x_1} + \frac{1}{x_2}\right)^{-\frac{1}{2}}$. The input prices for $x_1$ and $x_2$ are $w_1 = 4$ and $w_2 = 1$.

A) Show this function is increasing in $x_i$ for $x_1 > 0, x_2 > 0$.

$$\frac{\partial\left(\left(\frac{1}{x_1} + \frac{1}{x_2}\right)^{-\frac{1}{2}}\right)}{\partial x_1} = \frac{1}{2x_1^2\left(\frac{1}{x_2} + \frac{1}{x_1}\right)^{3/2}}$$

*Since $x_1, x_2 > 0$, the denominator is strictly positive.*

B) What is the degree of homogeneity for this function?

$$\frac{1}{2}$$

C) What are the firm's conditional factor demands?

$$x_2 = 3y^2$$

$$x_1 = \frac{3}{2}y^2$$

D) What is the firm's cost function.

$$9y^2$$

E) How much output does a monopolist produce? What does it charge?

$$y = \frac{100}{20} =$$

F) How much does *each firm produce* in Cournot oligopoly with $N$ firms?

$$\frac{\partial\left((100 - q_i - q_{-i})\, q_i - 9q_i^2\right)}{\partial q_i} = 100 - q_{-i} == 20q_i$$

$$y_i = \frac{100}{N + 19}$$

G) The government really wants to have 81 firms in this market for some reason. They decide to charge a license fee $F$ to any firm operating in the market. What is an amount $F$ the government can set so that 81 firms enter the market?

$$F = 10$$

2. A consumer has utility function $u = ln\left(x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}\right) + x_3$ and income $m$. Prices are $p_1, p_2, p_3$. Assume $p_3 = 1$ throughout the problem.

$$\frac{\partial\left(\log\left(x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}\right) + x_3 - \lambda\left(x_1 p_1 + x_2 p_2 + x_3 - m\right)\right)}{\partial x_1} = \frac{1}{2x_1} - \lambda p_1$$

$$\frac{\partial\left(\log\left(x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}\right) + x_3 - \lambda\left(x_1 p_1 + x_2 p_2 + x_3 - m\right)\right)}{\partial x_2} = \frac{1}{2x_2} - \lambda p_2$$

$$\frac{\partial\left(\log\left(x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}\right) + x_3 - \lambda\left(x_1 p_1 + x_2 p_2 + x_3 - m\right)\right)}{\partial x_3} = 1 - \lambda$$

If $m > 1$

$$\frac{1}{2p_1} = x_1$$

$$\frac{1}{2p_2} = x_2$$

$$x_3 = m - 1$$

If $m < 1$

$$x_1 = \frac{\frac{1}{2}m}{p_1}$$

$$x_2 = \frac{\frac{1}{2}m}{p_2}$$

A) Show that this function is concave. Do this however you'd like.

B) Set up the Lagrangian from which the unconstrained maximum gives the consumer's constrained utility maximum. Use $\lambda$ for the multiplier.

C) Using the derivatives of the Lagrangian, argue that: If the consumer is optimizing and consumes some of each good, the consumer's marginal utility for money is 1.

D) What are this consumer's demands for $x_1, x_2$ and $x_3$? Account for any corner solutions.

E) For what range of income will the consumer choose positive $x_1$? What about $x_2$? What about $x_3$?

F) Write the consumer's value function for the range of income where demand is strictly positive for all goods.

G) Using duality, write the consumer's expenditure function for the range of utilities where the consumer demands positive amounts of both goods.

H) What is the consumer's Hicksian demand for $x_1$ when utility is such that all goods are demanded?

3. $\succsim$ is a binary relation that is complete and transitive on some finite set $X$. Let $\succ$ be defined as follows: $x \succ y \Leftrightarrow x \succsim y$ and not $y \succsim x$. **Show that $x \succ y$ and $y \succsim z$ implies $x \succ z$.**

We need to prove not $z \succsim x$. Suppose otherwise.

$y \succsim z, z \succsim x$ implies $y \succsim x$ which is contradictory