1. In a 2013 paper, Eugenia Cheng derived the formula for the ratio of topping to dough in the median bite of a pizza of radius r. Where t is the volume of topping and d is the volume of dough, this ratio is[1]:

$$\rho = \frac{t}{d}\frac{r^6}{\left(r^3 - 8\right)^2}$$

This formula assumes a pizza bigger than 2 inches in radius, so *assume $r > 2$ for the rest of this problem. Further, because your oven has a limited size, the biggest pizza possible is $r = 100$.*

Total pizza volume is $v = t + d$ where t and d are measured in cups. Suppose your utility over pizzas (ρ, v) is:

$$u\left(v, \rho\right) = v\left(1 - \left(\frac{243}{676} - \rho\right)^2\right)$$

A) Show ρ is quasi-convex in r for $r > 2$.

B) Suppose you have to make a pizza consisting of one cup of toppings $t = 1$ and three cups of dough $d = 3$. What is the size of the optimal pizza?

C) Suppose you have to make a pizza consisting of one cups of topping $t = 1$ and two cups of dough, $d = 2$. What is the size of the optimal pizza?

2. You are running an experiment to learn about preferences on the set $X = \{(2,0), (0,2), (4,0), (0,4), (1,1), (2,2)\}$. You assume preferences are reflexive, complete, transitive, and anti-symmetric (recall that this means all preferences are strict on distinct bundles).

A) If you assume preferences are also **strictly monotonic**, what can you infer about $\succ$ on X without observing any choices?

In your experiment, you first ask the subject to choose a bundle from the set $\{(2,0), (0,2), (1,1)\}$.

B) If you assume preferences are also **convex** and **strictly monotonic** and you observe a subject choose $(2,0)$ from the set $\{(2,0), (0,2), (1,1)\}$, what can you infer about $\succ$?

C) If you assume preferences are also **homothetic**, **convex** and **strictly monotonic** and you observe a subject choose $(2,0)$ from the set $\{(2,0), (0,2), (1,1)\}$, what can you infer about $\succ$?

D) What are two additional sets can you ask this subject (from part **c**) to choose from so that you will be able to infer their **entire rank ordering**?

[1] I have adjusted this formula slightly for the problem.

3. A consumer has utility function $u = (x_1 + 1)(x_2 + 1)$ and income m. Prices are p_1, p_2. For each part, account for corner solutions.

A) Show that u is quasi-concave.

B) What is the consumer's Marshallian demand?

C) What is the consumer's elasticity of demand for x_1?

D) What is the consumer's Hicksian demand?