1.A. To show the function is quasi-convex, it is sufficient to show it is monotonic. This is the case for $r > 2$.

$$\frac{\partial\left(\frac{r^6}{(r^3-8)^2}\right)}{\partial r} = \frac{6r^5}{\left(r^3-8\right)^2} - \frac{6r^8}{\left(r^3-8\right)^3}$$

$$\frac{6r^5}{\left(r^3-8\right)^2} - \frac{6r^8}{\left(r^3-8\right)^3} < 0$$

$$r > 2$$

1.B. Notice that the utility is decreasing in the square of the term $\frac{243}{676} - \frac{1}{3}\frac{r^6}{(r^3-8)^2}$ and this term can be made zero, which is necessary (and sufficient) for maximization.

$$\frac{243}{676} - \frac{1}{3}\frac{r^6}{\left(r^3-8\right)^2} = 0$$

$$r = 6$$

1.C. Now the relevant cannot be made to be zero since $\frac{1}{2}\frac{r^6}{(r^3-8)^2}$ is always larger than $\frac{1}{2}$. Instead, we need to minimize this term:

$$\left(\frac{243}{676} - \frac{1}{2}\frac{r^6}{\left(r^3-8\right)^2}\right)^2$$

Looking for where this is stationary will fail. That's because it is decreasing above $r > 2$. So, the optimal is just to choose the largest r possible $r = 100$

2.A. We get a lot from strict monotonicity here. $(4,0) \succ (2,0), (0,4) \succ (0,2), (2,2) \succ (1,1), (2,2) \succ (0,2), (2,2) \succ (2,0)$.

2.B. From the choice, we can infer that $(2,0) \succ (0,2)$ and $(2,0) \succ (1,1)$. In addition to these and the inferences from monotonicity, we can now also infer $(1,1) \succ (0,2)$ by convexity.

2.C. By Homotheticity, we can additionally infer that $(4,0) \succ (0,4), (4,0) \succ (2,2), (2,2) \succ (0,4)$

2.D. At this point the only comparison that is left is $\{(0,4), (2,0)\}$ and $\{(0,4), (1,1)\}$

3.A. $(x_1+1)(x_2+1) = e^{(ln((x_1+1)(x_2+1)))} = e^{ln(x_1+1)+ln(x_2+1)}$ this is a monotonic transformation of $ln(x_1+1) + ln(x_2+1)$ which is a sum of concave functions and thus concave. Since $(x_1+1)(x_2+1)$ is a monotonic transformation of this, it is quasi-concave.

3.B. $x_1 = \frac{m-p_1+p_2}{2p_1}, x_2 = \frac{m+p_1-p_2}{2p_2}$ if $m \geq p_2 - p_1$ and $m \geq p_1 - p_2$.

Notice that m has to be greater than at least one of $m \geq p_2 - p_1$ and $m \geq p_1 - p_2$. But if either fails, we hit a corner.

$x_1 = 0, x_2 = \frac{m}{p_2}$ if $m < p_2 - p_1$

$x_1 = \frac{m}{p_1}, x_2 = 0$ if $m < p_1 - p_2$

3.C. $\frac{\partial\left(\frac{m-p_1+p_2}{2p_1}\right)}{\partial p_1} \frac{p_1}{\frac{m-p_1+p_2}{2p_1}} = \frac{2p_1^2\left(-\frac{m-p_1+p_2}{2p_1^2} - \frac{1}{2p_1}\right)}{m-p_1+p_2}$ if $m \geq p_2 - p_1$ and $m \geq p_1 - p_2$.

-1 if $m < p_1 - p_2$

0 if $m < p_2 - p_1$

3.D.

$x_1 = \frac{\sqrt{p_2}\sqrt{u}-\sqrt{p_1}}{\sqrt{p_1}}, x_2 = \frac{\sqrt{p_1}\sqrt{u}-\sqrt{p_2}}{\sqrt{p_2}}$ if $u \geq \frac{p_2}{p_1}$ and $u \geq \frac{p_1}{p_2}$. Notice that $u \geq 1$ for any (x_1, x_2). Thus, only one of the two of these can fail.

$x_1 = 0, x_2 = u - 1$ if $u < \frac{p_2}{p_1}$

$x_1 = u - 1, x_2 = 0$ if $u > \frac{p_1}{p_2}$