

## 8100 Problem Set 3.

October 5, 2021

1. Find the bundle that maximizes  $U(x) = x_1x_2$  subject to the two constraints: **(1)**  $(x_1^2 + x_2^2)^{\frac{1}{2}} \leq 10$  and **(2)**  $2x_1 + x_2 \leq 20$ . What constraints bind at this point? What is the value of the Lagrange multiplier at the optimum for the constraint(s) that bind?
2. When maximizing  $U(x) = x_1x_2$  subject to the two constraints: **(1)**  $(x_1^2 + x_2^2)^{\frac{1}{2}} \leq 10$  and **(2)**  $2x_1 + x_2 \leq m$ . Find a condition on  $m$ , such that only constraint 1 binds at the optimum.
3. Prove that for two budgets  $p_1x_1 + p_2x_2 \leq m$  and  $\tilde{p}_1x_1 + \tilde{p}_2x_2 \leq \tilde{m}$  and bundle available in the convex combination:  $(tp_1 + (1-t)\tilde{p}_1)x_1 + (tp_2 + (1-t)\tilde{p}_2)x_2 \leq tm + (1-t)\tilde{m}$  is affordable in one of the two original budgets.
4. Maximize  $u(x_1, x_2) = x_1 + \ln(x_2)$  subject to  $p_1x_1 + p_2x_2 = m$ . Set up the Lagrange function and find the first order condition. Interpret this condition in economic terms. At what price income combinations is the slope of the indifference curve at the optimum bundle not equal to the slope of the boundary of the budget set? Using your interpretation of the first order condition, say something about why these price/income combinations lead to a situation where the equal slope condition cannot be met.