# Competing for priorities in school choice

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**Abstract**

We present a model in which students can influence their priorities in a school choice mechanism through a first-stage costly effort game. We show that efficiency improvements to the mechanism can lead to net efficiency losses if they come at the price of increased allocative inequalities, increasing competition in the effort stage. We apply these results to the deferred and immediate acceptance mechanisms ($DA$ and $IA$) and show that, even when $DA$ is more allocatively efficient than $IA$, $IA$ may remain more efficient overall because it features less inequality between students with high and low priorities.

## 1 Introduction

Students in New York City who want to attend one of the City's most prestigious high schools such as Stuyvesant or Bronx Science must take the Specialized High School Admissions Test. This test's results influence placements by determining students' priorities in the subsequent mechanism that assigns students to schools. Students who score higher on the placement exam receive higher priority in the procedure, and a higher priority tends to yield a seat in a more preferred school. Since the Specialized Schools use a sequential dictator mechanism, the top-scoring student receives her top choice. The one-hundredth best scoring student only receives a seat in her most favored school among those that the ninety-nine top-scoring students have not already filled (Dobbie and Roland G. Fryer, 2011).

In other places like Boston, students acquire a school priority based on several criteria, such as whether the student lives within walking distance to the school (Abdulkadiroğlu et al., 2005b). This implies that students compete for access to better schools, in part, by relocating near their favorite schools, as evidenced by the effect of school quality on house prices (Crone, 1998; Downes and Zabel, 2002; Leech and Campos, 2003; Reback, 2005; Gibbons and Machin, 2008). Again, higher priorities at better schools are useful only as far as the assignment mechanism uses these priorities. In Boston, for example, the use, since 2005, of a *deferred acceptance* mechanism ($DA$) gives a lot of importance to priority, which contrasts with the previous use of an *immediate acceptance* mechanism ($IA$) where priority plays a lesser role.

The matching mechanism itself is an essential component in the overall assignment process. The matching literature has made significant progress in analyzing the properties of mechanisms. Abdulkadiroglu and Sönmez (2003) and Pathak (2011) provide a review of results in school choice.[1] Famously, these results led to the redesign mentioned above for the assignment procedure in Boston, and the redesign of the broader New York public school assignment procedures in 2003 (Abdulkadiroğlu et al., 2005). Researchers generally analyze mechanisms by assuming that priorities are fixed and exogenous. *In this paper, we emphasize that the interaction between the matching mechanism and the "game" through which priorities are determined can be an important consideration, in particular for the overall efficiency of the assignment procedure.*

To illustrate the importance of this interaction, suppose that a school district allocates $x\%$ of its seats through an algorithm that respects priorities (such as $DA$) and allocates the remaining $(100-x)\%$ of seats through a random serial dictatorship, the order of which is *independent of priorities*.[2] Further, suppose that priorities at a school are a continuous function of how close a student lives to the school. This is the case, for instance, in the French-speaking region of Belgium, where one of the factors affecting priority is the distance between one's residence and the school at stake (Cantillon, 2015).

Clearly, as $x$ increases, incentives to relocate closer to one's favorite schools increase.[3] Furthermore, to the extent that relocation efforts are wasteful,[4] welfare could be negatively affected by an increase in $x$.

Of course, the overall efficiency picture is more complicated. In particular, the effect of a change in mechanism on the wasteful competition for priorities must be weighed against the possible gains in allocative efficiency brought about by that change.[5] To study overall efficiency, we embed a simplified matching environment into a two-stage game with a first-round contest for priority (the "*effort stage*"). This allows us to formally analyze the effect of changes to the matching mechanism in a "general equilibrium" framework, which includes interactions between the mechanism and preliminary effort.

This approach yields important insights. We show that efficiency improvements at the level of the mechanism can lead to net efficiency *losses* through increased competition in the effort stage (by "net" efficiency, we mean efficiency at the level of the procedure as a whole, including the effort stage). All else equal, net efficiency increases with the efficiency of the mechanism but decreases with its allocative inequality because increases in the latter induce fiercer competition — and higher wasteful effort — at the level of the effort stage. Decreasing allocative *inequality* at the level of the mechanism can therefore be justified by net *efficiency* considerations. Improvements in allocative efficiency can also have detrimental *net* efficiency effect if they come at the price of increased allocative inequalities. In the context of our simplified matching environment, this implies that even when $DA$ is more allocatively efficient than $IA$, the latter may remain more efficient

[1] See Sönmez and Ünver (2011) for a broader overview.

[2] In practice, it is common for school districts to allocate different layers of seats — or seats at different schools — through different mechanisms. Examples include New York City, which uses a different mechanism for elite high-schools than for the remaining high schools in the district (Abdulkadiroğlu et al., 2005a; Dobbie and Roland G. Fryer, 2011).

[3] Reback (2005) show empirically that a change in the public school assignment procedure in Minnesota in the early '90s (namely the adoption of a public school choice program) led to significant changes in home prices. Priority access to schools caused these price movements.

[4] Moving closer to one's favorite schools may imply choosing a house and neighborhood that would not be one's optimal choice was it not for the quality of the schools in that neighborhood.

[5] In the above example, if increasing $x$ also makes the allocation of seats more efficient, this increase in efficiency may overwhelm the decrease in efficiency caused by fiercer competition for priorities.

overall because it features less inequality between students with high and low priority (in equilibrium).

While our results apply specifically to school choice and the $IA$ and $DA$ mechanisms, they point, more broadly, to the importance of considering matching mechanisms in a general equilibrium setting when costly effort is involved in influencing the potential outcomes.

## 1.1 Related literature

Our paper is the first, to our knowledge, to consider the effect of a matching mechanism on the incentives for players to compete for priority. However, several papers are related in terms of our general approach[6]

Hatfield et al. (2016) considers a general matching/allocation environment where players make investment decisions that impact their valuations in the matching stage (with potential uncertainty about the impact of investment). As in our paper, the authors consider the matching mechanism's effect on overall efficiency, including pre-matching investment. They demonstrate that a mechanism incentivizes efficient investment if and only if it is allocatively efficient and strategy-proof, generalizing previous results of Rogerson (1992) and Bergemann and Välimäki (2002). In section 4.2, we also consider the case where a player's investments affect their preferences. However, in contrast to Hatfield et al. (2016) we do not allow for transfers, and our players' investments affect how the allocation mechanism treats them.

The role of investments in Hsu (2016) mirrors our approach. There, students are endowed with an allotment of time to devote to studying various topics. These investments determine a student's priority in the matching stage, but not their valuation. In contrast to our paper, Hsu (2016) analyzes the effect of the mechanism choice on students' incentives to spend time on topics they have a particular talent for or to pursue a diversity of ability. We focus on a more traditional welfare analysis.

Hatfield et al. (2011) is closely related in theme to our work but takes a different approach to analyze competition. In their model, investments by schools affect students' preferences. However, instead of studying the welfare effect of investments as we do, they introduce an ordinal criterion that implies schools have an incentive to invest in improvements. They demonstrate that, in large school districts, this condition is approximately met when the matching mechanism is stable.

A final related paper is Zhang (2014), which demonstrates that stable two-sided matching mechanisms can induce players to engage in risky gambles for desirable characteristics in a pre-matching stage.

## 1.2 Paper structure

Our paper is structured as follows. In section 2, we describe our model and provide the details of the matching environment and pre-matching effort stage. In section 3, we demonstrate that the effort stage constitutes a *generic* all-pay contest and leverage

[6] Students' behavior in our model, which leads to decreases in net welfare despite improvements to the allocation of schools, which is a specific case of rent-seeking. See, for instance, Tullock (1967); Krueger (1974); Buchanan et al. (1980); Tullock et al. (1993); Congleton et al. (2008); Tollison (2012).

the results of Siegel (2009) to derive comparative statics which relate properties of the matching mechanism to overall welfare in the equilibrium of the two-stage effort/matching game. In section 4 we use these results to study the overall efficiency of popular school choice mechanisms such as *deferred acceptance* and *immediate acceptance* under various conditions. We conclude with a discussion of our results and potential extensions in section 5.

# 2 Model

Our model is split into two main stages. In the effort stage, students compete for "priorities" in a contest. In the second stage, these priorities are used in a school choice mechanism to determine seats at schools. The timing of our model is as follows: students first compete in a contest by simultaneously choosing effort under common knowledge of effort costs. Students then learn their rank in the contest and how these ranks translate into priorities at schools. Students also learn their cardinal preferences over schools (but not the cardinal utility of other students) before reporting ordinal preferences over schools in the second stage school choice mechanism. We discuss some aspects of the timing and information structure further in section 2.3. Since the effort stage's incentives depend on the matching mechanism's potential outcomes, it is convenient to describe the matching stage first.

## 2.1 Matching stage

The model of the matching stage is similar to Abdulkadiroğlu et al. (2011). There are $m \geq 2$ schools, $S := \{s_1, \dots, s_m\}$ with the index set $A := \{1, \dots, m\}$. School $s_a \in S$ has capacity $q_a > 0$. Departing from Abdulkadiroğlu et al. (2011), we divide schools into two groups. The set of "high-quality" schools is $S^h := \{s_1, \dots, s_g\}$ for some $g \in \{1, \dots, m-1\}$ and the set of "low-quality" schools is $S^\ell := \{s_{g+1}, \dots, s_m\}$.

Schools in $S$ can be attended by $n \geq 2$ students. Among these students, $h$ have a high **priority-type** which results in a high priority at schools $s_1$ to $s_g$ (these are the $h$ students who exert the highest effort in the initial contest stage, see below).[7] Priorities are otherwise determined via a symmetric tie-breaking rule.[8] This simple priority structure can be viewed as an approximation of school districts where only a few strict priorities are awarded (e.g., based on whether a student lives in the "walking zone" of a school) with ties otherwise broken at random, which is common in practice. Throughout, we assume that $\sum_{a\in\{1,\dots,g\}} q_a = h$ and $\sum_{a\in\{g+1,\dots,m\}} q_a = \ell$. That is, the total capacity of the

[7] Slightly abusing the notation, we use $\ell$ and $h$ to denote both the priority-types themselves (low or high) and the number of students of a certain priority-type ($\ell$ low priority-types and $h$ high priority-types).

[8] This means the $h$ high priority-types always have a higher priority at the $g$ high-quality schools than the $\ell$ low priority-types. Priorities at the high-quality schools among the $h$ high priority-types and the $\ell$ low priority-types are determined symmetrically at random (where "symmetrically" we mean that each student has the same probability of occupying any priority-rank at a high-quality school than any other student of her priority-type). Priorities at the low-quality schools are also determined symmetrically at random but among *all* students, low and high priority-types alike.

high-quality schools is equal to the number of high priority-type students and the total capacity of the low-quality schools is equal to the number of low priority-type students.

Each student draws vNM utility values $\boldsymbol{v} = (v_1, \ldots, v_m)$ about the schools with probability $f_p(\boldsymbol{v})$, where the distribution $f_p(\boldsymbol{v})$ may depend on the students' priority-type $p \in \{\ell, h\}$.[9] We refer to $\boldsymbol{v}$ as a student's **valuation-type**. Students compete for higher priorities through the effort stage described below. *After* acquiring a certain priority-type through the effort stage, students learn their valuation-type (but not the valuation-type of other students) and report ordinal preferences over schools. Assignment to a school is determined based on priorities and reported preferences by some school choice mechanism $X$. This paper focuses on the deferred acceptance ($DA$) and immediate acceptance mechanisms ($IA$). We refer to the extensive school choice literature for a detailed description of these mechanisms (e.g., Abdulkadiroglu and Sönmez, 2003).

Students learn their utility value *after* the effort stage (see section 2.3). Therefore, at the level of the effort stage, students have identical ex-ante utilities for securing a high or a low priority-type. We denote these ex-ante utilities by $V_h$ and $V_\ell$, or sometimes $V_h^X$ and $V_\ell^X$ when a reference is made to the mechanism $X$ the equilibrium of which induces utilities $V_h$ and $V_\ell$. Expected utilities $V_\ell$ and $V_h$ are the utility students base their decisions on in the effort stage. For consistency with the effort stage and to avoid possible extreme cases, we always assume $V_h \geq V_\ell$.

## 2.2 Effort stage

Students compete by putting effort into acquiring the characteristics that result in higher priorities. In the context of school choice, this may mean moving closer to a particular set of schools or spending time and money in preparing for a standardized test.

The **total effort** student $i$ puts into obtaining a higher priority is $t_i$. Student $i$'s total effort is decomposed into $i$'s **priority-*in*dependent effort** $a_i$ and $i$'s **priority-dependent effort** $r_i$. As its name suggests, we think of $a_i$ as $i$'s optimal effort given all benefits from effort independent of acquiring a higher matching priority. For example, $a_i$ could represent the amount of effort student $i$ would put into preparing for standardized tests "anyways", for *reasons other than* the effect of test-results on priorities at schools (e.g., social prestige, curiosity, and use in determining outcomes independent of school assignments). Priority-independent effort can also represent the effort a family would typically put into moving to a new neighborhood for *all reasons other than* a boost in priority at the schools in this neighborhood. In contrast, **priority-dependent effort** $r_i$ is the part of the total effort exerted because of the effect of effort on school-priorities. For example, $r_i$ could represent the extra-cost a family is willing to pay to relocate to a new neighborhood with desirable schools *specifically because of the higher priority* the family would secure at these desirable schools.

Because priority-*in*dependent effort is chosen optimally for reasons other than securing higher priorities, we take $a_i$ as given for every student and model it as a sunk cost. In our effort stage, students choose a level of total effort $t_i$ which is at least $a_i$. Whereas priority-dependent effort is a decision variable, priority-independent effort is an exogenous characteristic. Therefore, we often refer to a student's priority-independent effort as her

[9] We later impose constraints on the support of $f_p(\boldsymbol{v})$ that formalize the idea that schools in $\{s_1, \ldots, s_g\}$ are of higher quality than schools in $\{s_{g+1}, \ldots, s_m\}$.

| Types | Notation | Description | Timing |
|---|---|---|---|
| Effort-type | $a$ | Baseline effort a student exerts regardless of its effect on priority-types | Exogenously given at the beginning of the effort stage |
| Priority-type | $p$ | Whether the student is high or low priority at high-quality schools | Acquired through the effort stage |
| Valuation-type | $\boldsymbol{v}$ | Utility value of being assigned to a high-quality school | Drawn from distribution $f_p(\boldsymbol{v})$ after the effort stage |

Table 1: Summary of the three type-dimensions that characterize students through the game and when students learn about them.

**effort-type**. A summary of the three type-dimensions that characterize students through the game (i.e., effort-type, priority-type, and valuation-type) is provided in Table 1. Index players by their effort-type: $a_1 \geq a_2 \geq a_3 ... \geq a_n$. We assume that $a_{h+1} \neq a_j$ for $j \neq h+1$.[10]

Higher effort is costly and induces no benefit other than the higher priorities it may provide (recall that any "intrinsic value" of effort is already captured by $a_i$). The cost of total effort $t_i$ for student $i$ is given by $e(t_i)$. We assume that $e$ is continuous and strictly increasing. The cost of additional priority-dependent effort is $e(t_i) - e(a_i)$ which can also be written $e(r_i + a_i) - e(a_i)$.

If a student exerts one of the $h$ highest total efforts, she becomes a high-priority student and her utility is $V_h - (e(t_i) - e(a_i))$. Otherwise, she is a low priority student and her utility is $V_\ell - (e(t_i) - e(a_i))$. Given a vector of total effort $\boldsymbol{t}$, let $H_i(\boldsymbol{t})$ be probability student $i$ becomes high-priority. Define $H_i$ as follows: $\sum_{i=1}^n H_i(\boldsymbol{t}) = h$ and,

$$H_i(\boldsymbol{t}) = \begin{cases} 0 & t_i < t_j \text{ for at least } h \text{ players } j \neq i \\ 1 & t_i > t_j \text{ for at least } n-h \text{ players } j \neq i \\ \text{any value in } [0,1] & \text{otherwise} \end{cases}.$$

Student $i$'s utility given total effort vector $\boldsymbol{t}$ is:

$$u_i(\boldsymbol{t}) := H_i(\boldsymbol{t})\Big[V_h - \big(e(t_i) - e(a_i)\big)\Big] + \big(1 - H_i(\boldsymbol{t})\big)\Big[V_\ell - \big(e(t_i) - e(a_i)\big)\Big].$$

If $t^*$ is an equilibrium of the effort stage, the **net welfare** when ex-ante utilities are $V_\ell$ and $V_h$ and effort-types are $\boldsymbol{a}$ is

$$W(V_h, V_\ell, \boldsymbol{a}) := \sum_{i=1}^n u_i(\boldsymbol{t}^*).$$[11]

[10] This assumption is required for the model to meet the *power condition* of Siegel (2009).

[11] Dependence of $W(V_h, V_\ell, \boldsymbol{a})$ on a particular equilibrium $\boldsymbol{t}^*$ is omitted in the notation because we show below (Theorem 1) that equilibrium utilities are unique in the effort stage.

## 2.3 Discussion of model assumptions

At least two features of our model deserve comments. First, the game associated with the effort stage is one of complete information, whereas we model the matching stage as an incomplete information game. The latter is desirable as it is hard to conceive of realistic (especially large-scale) school choice problems where every student knows every other student's valuation-type. One could, however, argue that the same is true of effort-types.

As the equilibrium analysis reveals, coordination on a (complete information) Nash equilibrium in the effort stage requires much less than full knowledge of the effort-type profile. To select her equilibrium priority-dependent effort, a student only needs to know the *threshold effort* that is required to deter low effort-types from competing for higher priorities.[12] For example, if priority-types are awarded based on exam scores, knowing the threshold-scores that prevailed in previous years could provide a reasonably accurate estimate of the current year's threshold, and students might be able to coordinate on the Nash equilibrium.

However, in other situations, students do not have accurate information on the threshold effort and have to rely on stochastic beliefs about each other's effort-types to play the effort game.

In a companion paper (Leo and Van der Linden, 2018), we study an incomplete information variant of our complete information effort stage. There, we show that the main results under complete information have parallels under incomplete information with additional restrictions on the effort function. In particular, every result about the effort stage that is later used to compare the net welfare when $DA$ or $IA$ are used has an analog in the incomplete information setting (Leo and Van der Linden, 2018). Therefore, our net welfare comparisons of $DA$ and $IA$ extend to this incomplete information setting when these additional assumptions on the effort function are met.

Second, recall that students learn their valuation-type *after* learning the priority-type they acquired through the effort stage. The distribution from which valuations are drawn may depend on the acquired priority-type,[13] which students know. But all students compete for higher priorities before knowing the realization of these (possibly priority-type specific) distributions. In particular, students know they will prefer high-quality schools to low-quality ones, but *not* exactly *how intense* this preference will be.

Although students' *ordinal* ranking of schools may be relatively stable through time, students might indeed, in some cases, learn their precise *cardinal* values for schools after acquiring priorities. For example, this is the case; if to secure a higher priority at a neighborhood's school, students have to move to that neighborhood years in advance of applying to these schools.

More importantly, assuming that students have the same expectations about their future valuation-type in the effort stage, enables distinguishing between wasteful and signaling aspects of effort. As a first step and a benchmark, it can be useful to isolate the wasteful aspect of effort and compare net welfare under different school mechanisms

[12] In turn, determining the threshold effort requires knowing only the $h$-th highest effort-type, which is the only information on the effort-type profile that is required for students to coordinate on a (complete information) Nash equilibrium in the effort stage.

[13] E.g., if a student obtained a higher priority at a school by moving to the school's neighborhood, she may be drawing a utility value for that school from a different distribution than a student who did not move to the school's neighborhood (e.g., because she now lives closer to that school).

assuming that effort carries no welfare-relevant information. This can only be done if a higher effort does not signal a more intense preference for high-quality schools. This requires assuming that (i) in the matching stage, students draw valuations-types from the same distribution, regardless of the priority-type they acquired in the effort stage, and (ii) in the effort stage, students only know this distribution and do not have additional idiosyncratic information about their future valuation-type We maintain these two assumptions throughout section 4.1.

In section 4.2, we open the door to effort being informative of preference intensity by relaxing (i) and allowing the distribution of valuation-types to depend on priority-types. In this section, we *maintain* the assumption that students only learn the realization of their valuation-type *after* the effort stage (i.e., we essentially maintain (ii)). However, students now anticipate that if they acquire a high priority-type, they will draw a valuation-type from a different distribution than if they had acquired a low priority-type. As we show, this is sufficient for effort to become a welfare relevant signal and alter some — but not all — of the net welfare comparisons obtained in section 4.1. Relaxing (ii) and endowing students in the effort stage with idiosyncratic information about their future valuation-type may provide further opportunities for effort to become a signal of preference intensity. We discussed this last point in the Conclusion and the Appendix.

# 3 Equilibrium in the effort stage

After a normalization to measure utility relative to the value of being low-priority, this model is a *generic* all-pay contest (Siegel, 2009, see the Appendix, section A.1).[14]

## 3.1 Utility in equilibrium

Let $\Delta = V_h - V_\ell$ we refer to this as the *allocative inequality.* Let the *threshold score* of the contest be the $\tilde{t}$ that solves $\Delta = e\left(\tilde{t}\right) - e\left(a_{h+1}\right)$. Thus,

$$\tilde{t} = e^{-1}\left(\Delta + e\left(a_{h+1}\right)\right).$$

Since this contest is *generic*, the expected utilities of the players in any equilibrium of the first-stage effort game are characterized by Siegel (2009, Theorem 1).

**Theorem 1.** *In any equilibrium with effort-type vector* $\boldsymbol{a}$*, the expected utilities of a player with effort-type* $a_i$ *is:*

$$u_i(\boldsymbol{a}) \coloneqq \begin{cases} V_h, & \text{if } i < h+1 \text{ and } a_i > e^{-1}(\Delta + e(a_{h+1})) \\ V_\ell + e(a_i) - e(a_{h+1}), & \text{if } i < h+1 \text{ and } a_i < e^{-1}(\Delta + e(a_{h+1})) \\ V_\ell, & \text{if } i \geq h+1 \end{cases}. \quad (1)$$

In any equilibrium, the $n-h$ players with the lowest baseline scores exert no priority-dependent effort and have an expected utility of $V_\ell$.[15] This is precisely what they would expect to get if the baseline scores themselves were used to determine priority.

[14]This model meets assumptions *A1-A3* along with the *Power* and *Cost* conditions of Siegel (2009).

[15] This is in terms of un-normalized utility.

The players with the $h$ highest baseline scores, on the other hand, have expected utility equal to $V_h$ less the minimum effort cost of obtaining at least the threshold score. Whether these players exert priority-dependent effort depends on whether or not their baseline score is already above the threshold. We say that players for which $i < h+1$ and this is not the case are in the **competitive set**, which we denote by $C(\Delta, \boldsymbol{a})$ . Formally $C(\Delta, \boldsymbol{a})$ is the set of students $i \in 1, ..., h$ such that $\Delta + e(a_{h+1}) \geq e(a_i)$, which corresponds to the second line of (1). Let $\#C(\Delta, \boldsymbol{a})$ be the size of this set. For given $e$ and $\Delta$, we say that $\boldsymbol{a}$ is **more competitive** the larger $\#C(\Delta, \boldsymbol{a})$.

With the utility characterization in Theorem 1, the net welfare in any equilibrium also has a simple characterization. We first define a few terms. As defined earlier, $\Delta$ is the *allocative inequality*, the difference between the expected utility of high and low priority players in the second stage. $V$ is the *allocative welfare*. It corresponds to the welfare in a model where additional effort is not possible and baseline scores are used to determine priority. Finally, we define the *effort deadweight loss* $D(\Delta, \boldsymbol{a})$. This is the aggregate welfare cost of effort expended in the effort stage:

**Allocative Inequality:** $\Delta := V_h - V_\ell$

**Allocative Welfare:** $V := hV_h + (n-h)V_\ell = nV_\ell + h\Delta$

**Effort Deadweight Loss:** $D(\Delta, \boldsymbol{a}) = \sum_{i=1}^{h} max\left\{\Delta + e(a_{h+1}) - e(a_i), 0\right\}$

We then have the following characterization.

**Corollary 1.** *In any equilibrium of the effort stage, expected* net welfare *is* allocative welfare *less the* effort deadweight loss*, i.e.,*

$$W(V_h, V_\ell, \boldsymbol{a}) = V - D(\Delta, \boldsymbol{a}).$$

## 3.2 Comparative statics

Allocative welfares $V_\ell$ and $V_h$ affect net welfare both directly through allocative welfare and indirectly through effort deadweight loss due to changes in competitive incentives. Increases to $V_\ell$ strictly increase allocative welfare. Increases to $V_\ell$ also reduce the relative "prize" earned in becoming high-priority. This reduces competitive incentives, weakly decreasing effort deadweight loss. Thus, changes to $V_\ell$ strictly increase net welfare.

While increasing $V_h$ also improves allocative welfare, this increase also changes incentives to compete, weakly increasing effort deadweight loss. However, this effect never overwhelms improvements in allocative welfare, and increases in $V_h$ weakly increase net welfare. The improvement is strict as long as some player does not need to put in any extra effort beyond her priority-independent effort to reach the threshold score. That is, if there is a student for which $\Delta + e(a_{h+1}) < e(a_i)$. For such a student, priority-independent effort is already enough to guarantee high priority in equilibrium since there are not $h$ other students willing to put in such a level of effort even to attain high-priority with certainty.

Overall, the magnitude of the effect of $V_h$ and $V_\ell$ on welfare is mediated by the size of the competitive set. The larger the competitive set, the more net welfare increases as $V_\ell$ increases, and the less it increases as $V_h$ increases. When the competitive set is the entire set of top $h$ students (indexed by priority-independent effort), $V_h$ does not affect net welfare.

All Propositions and Corollaries in this section follow straightforwardly from Theorem 1, and proofs are therefore omitted.

**Proposition 1.** *For almost all values of* $V_h, V_\ell, \boldsymbol{a}$*,*

$$\frac{\partial W\left(V_h, V_\ell, \boldsymbol{a}\right)}{\partial V_h} = h - \#C(\Delta, \boldsymbol{a}), \quad \frac{\partial W\left(V_h, V_\ell, \boldsymbol{a}\right)}{\partial V_\ell} = (n-h) + \#C(\Delta, \boldsymbol{a}).$$

Since players are indexed by $a_i$ and $e$ is strictly increasing, all $i < h+1$ are in the competitive set as long as player 1 is in the competitive set. This leads to the following corollary:

**Corollary 2.** *For any* $V_h, V_\ell, \boldsymbol{a}$*, net welfare* $W(V_h, V_\ell, \boldsymbol{a})$ *is strictly increasing in* $V_\ell$*. It is increasing in* $V_h$*; strictly so if and only if* $\Delta + e\left(a_{h+1}\right) < e\left(a_1\right)$*.*

On the other hand, if even the student with the highest priority-independent effort is in the competitive set, improvements to $V_h$ do not affect welfare since direct improvements to allocative welfare are completely offset by increased competitive incentives.

**Lemma 1.** *For any* $V_h, V_\ell, \boldsymbol{a}$*, if* $\Delta + e\left(a_{h+1}\right) \geq e\left(a_1\right)$*, then* $W(V_h, V_\ell, \boldsymbol{a}) = nV_\ell$ *and* $\frac{\partial W(V_h, V_\ell, \boldsymbol{a})}{\partial V_h} = 0$

In our context, Corollary 1 provides and efficiency justification for fairness considerations. Changes to the allocation mechanism that do not weakly improve the low types' outcome cannot provide efficiency gains that are robust in highly competitive environments. In other words, for any change to the allocation mechanism that lowers $V_\ell$, there is an environment that is sufficiently competitive for that change to lower net welfare. This is true even if the change increases allocative welfare. For example, it may be tempting to increase $V_h$ by a large amount at the expense of a small decrease in $V_\ell$. However, in very competitive environments, the high ability types would compete away almost all of this efficiency improvement such that even a small decrease in value to the low priority players is enough to cause a net loss in welfare.

This logic is formalized in the following proposition. Again, it shows that an improvement in the welfare of the low priority-type is essential to any change to the allocation mechanism aimed at *robustly* improving net welfare (where robustness is with respect to the competitiveness of the effort stage).

**Proposition 2.** *For any* $V_h, V_\ell, \boldsymbol{a}$ *and* $V_h', V_\ell'$ *such that* $V_h > V_h'$ *and* $V_\ell < V_\ell'$*, there exists an* $\boldsymbol{a}'$ *such that* $W(V_h, V_\ell, \boldsymbol{a}) < W(V_h', V_\ell', \boldsymbol{a}')$*.*

The previous results emphasizes the importance (from an efficiency perspective) of the fate of the low types as competitiveness increases. Even for a fixed, arbitrary level of competitiveness, inequalities between the low and the high types (as represented by $\Delta$) have a significant impact on efficiency. Similarly, fixing allocative welfare, an increase in inequality harms welfare as long as the competitive set is not empty.

**Proposition 3.** *For any pair of* $V_h, V_\ell$ *and* $V_h', V_\ell'$ *and any* $a$ *such that* $V(V_h, V_\ell) = V(V_h', V_\ell')$*,* $\Delta' > \Delta$*, and where* $\Delta' + e\left(a_{h+1}\right) > e\left(a_h\right)$*,* $W(V_h, V_\ell, \boldsymbol{a}) > W(V_h', V_\ell', \boldsymbol{a})$*.*

Naturally, although Proposition 3 isolates the inequality effect by imposing $V(V_h, V_\ell) = V(V'_h, V'_\ell)$, the proposition also has implications for situations where efficiency is improved. Specifically, any efficiency improvement can be off-set by a sufficient increase in inequalities.

**Corollary 3.** *For any pair of $V_h, V_\ell$ and any $\epsilon > 0$, there exists $V'_h, V'_\ell$ such that $V(V'_h, V'_\ell) = V(V_h, V_\ell) + \epsilon$ but $\Delta' > \Delta$ and $W(V'_h, V'_\ell, \boldsymbol{a}) < W(V_h, V_\ell, \boldsymbol{a})$.*

# 4 Equilibrium in the matching stage

When reporting her preference, a student knows her priority-type $p$ as well as her own valuation-type $\boldsymbol{v}$, but not the valuation-type of other students. The equilibrium probability for a priority-type $p$ to be assigned to school $s_a$ when the mechanism is $X$ is denoted by $P_a^{X,p}$. In equilibrium, the expected utility of such a student is then

$$V_{p,\boldsymbol{v}}^X := \sum_{a \in A} v_a P_a^{X,p}.$$

Recall that students learn the realization of their valuation-type "right before" submitting their preference. In particular, students do not know the realization of their valuation-type during the effort stage. In the effort stage, students only know the distribution $f_p(\boldsymbol{v})$ from which they will draw their valuation-type (which, as the subscript indicates, may depend on the priority-type $p$ they acquire through the effort stage). Throughout we assume that the supports of $f_\ell$ and $f_h$ are finite, and we let $\mathcal{V}$ denote a generic set of valuation-types that includes the supports of both $f_\ell$ and $f_h$.[16] Before knowing the realization of $f_p(\boldsymbol{v})$, the expected utility of a priority-type $p$ in mechanism $X$ is then

$$V_p^X := \sum_{\boldsymbol{v} \in \mathcal{V}} V_{p,\boldsymbol{v}}^X f_p(\boldsymbol{v}).$$

In the effort stage, students base their decision on expected utilities $V_\ell^X$ and $V_h^X$. Recall that the allocative welfare of a given mechanism $X$ is then defined as $V^X = hV_h^X + (n - h)V_\ell^X$.

## 4.1 Priority-independent valuations

In this section, we follow Abdulkadiroğlu et al. (2011) in assuming that the support of $f$ is a finite set $\tilde{\mathcal{V}} \subset \{(v_1, \ldots, v_m) \in [0,1]^m \mid v_1 > v_2 > \cdots > v_m\}$. In particular, all students have the same ordinal preference, preferring school $s_a$ to school $s_b$ if $a < b$, which is conceivable in areas where schools have a clear quality ranking. Importantly, however, students may differ in their relative preference intensities. In this section, we also focus on priority-*in*dependent valuations. That is, every student draws a valuation-type $\boldsymbol{v}$ from the *same* distribution $f(\boldsymbol{v})$, regardless of whether the student is a high or a low priority-type (i.e., $f_\ell = f_h = f$). The case of priority-dependent valuations is treated in the next section.

[16] Finiteness is assumed to simplify the existence of Bayesian equilibria in $IA$.

Because $DA$ has a truthful dominant strategy, it makes sense to assume that individuals report preferences truthfully in $DA$. Thus, because students have the same ordinal preference over schools, every student reports the same ranking of schools in $DA$. In $DA$, the $h$ high priority-types are therefore assigned at random to the $h$ high-quality schools. The $\ell$ low priority-types are, on the other hand, assigned at random to the $\ell$ low-quality schools. Given the (dominant strategy) equilibrium of $DA$ we let $\hat{P}_a^p$ denote the probability that a student of type $p$ is assigned to school $s_a$, that is

$$\hat{P}_a^h = q_a/h \text{ if } a \in \{1, \dots, g\}, \text{ and 0 otherwise, and}$$
$$\hat{P}_a^\ell = q_a/\ell \text{ if } a \in \{g+1, \dots, m\}, \text{ and 0 otherwise.}$$

For a given (symmetric Bayesian) equilibrium $\sigma^*$ of $IA$ and any strategy $\sigma$, let $\dot{P}_a^p(\sigma)$ be the probability that a student with priority-type $p$ is assigned to school $s_a$ if that student plays strategy $\sigma$ and all other students play the symmetric equilibrium strategy $\sigma^*$.[17] An equilibrium $\sigma^*$ of $IA$ is **segregating** if high priority-types are never assigned to a low-quality school, i.e., $\sum_{\boldsymbol{v}\in\tilde{\mathcal{V}}} \dot{P}_y^h(\sigma_h^*(\boldsymbol{v}))f_h(\boldsymbol{v}) = 0$ for every low-quality school $s_y \in \{s_{g+1}, \dots, s_m\}$ (as in the dominant strategy equilibrium of $DA$). In contrast, an equilibrium $\sigma^*$ of $IA$ is **blending** if high priority-types are sometimes assigned to at least one low-quality school, i.e., $\sum_{\boldsymbol{v}\in\tilde{\mathcal{V}}} \dot{P}_y^h(\sigma_h^*(\boldsymbol{v}))f_h(\boldsymbol{v}) > 0$ for some low-quality school $s_y \in \{s_{g+1}, \dots, s_m\}$.

**Example 1** (Blending equilibria with $IA$)**.** Suppose there are three schools, two high-quality schools $s_{H1}$ and $s_{H2}$, and one low-quality school $s_L$. The distribution of valuations is the degenerate $f(\boldsymbol{v}^*) = 1$ with $v_{H1}^* > v_{H2}^* > v_L^*$. High-priority types can only rank one high-quality school first and there is always a high-quality school $H^*$ that less than $q_{H^*}$ high priority-types rank first. Therefore, low priority-types have a non-zero probability of being assigned to $s_{H^*}$ if they rank $s_{H^*}$ first. As a consequence, low priority-types always secure a payoff strictly larger than $v^*(s_L)$ in equilibrium. This, in turn, implies that the equilibrium of $IA$ must be blending because low priority-types would get a payoff of $v_L^*$ in a segregating equilibrium.

The next theorem shows that low priority-types always prefer the equilibrium outcome of $IA$ to the dominant strategy outcome of $DA$.

**Theorem 2.** *For any priority-independent distribution $f$, any valuation-type $\boldsymbol{v} \in \tilde{\mathcal{V}}$, and any symmetric equilibrium of $IA$, we have $V_{\ell,\boldsymbol{v}}^{IA} \geq V_{\ell,\boldsymbol{v}}^{DA}$, and $V_{\ell,\boldsymbol{v}}^{IA} > V_{\ell,\boldsymbol{v}}^{DA}$ if the equilibrium of $IA$ is blending.*

*Proof.* Let $\sigma^*$ be any symmetric equilibrium of $IA$, with $\sigma_\ell^*$ the strategy played by the low priority-types and $\sigma_h^*$ the strategy played by the high priority-types. For every school $s_a$, the feasibility constraint (respecting quotas at schools) and the fact that $IA$ is non-wasteful imply

$$\ell \sum_{\boldsymbol{v}\in\tilde{\mathcal{V}}} \dot{P}_a^\ell(\sigma_\ell^*)f(\boldsymbol{v}) + h \sum_{\boldsymbol{v}\in\tilde{\mathcal{V}}} \dot{P}_a^h(\sigma_h^*)f(\boldsymbol{v}) = q_a. \tag{2}$$

[17] A strategy $\sigma$ is a mapping from the set of valuation-types into the set of mixed strategies over reported preferences.

Equality (2) simply expresses the fact that in an equilibrium of $IA$, all the seats are distributed and feasibility constraints are respected.

Fix any valuation-type $\ddot{v} \in \tilde{\mathcal{V}}$. We show that when student $i$ with valuation-type $\ddot{v}$ is a low priority-type, (2) implies that if other students play the equilibrium strategies specified by $\sigma^*$, student $i$ can always play a strategy $\tilde{\sigma}$ that makes her at least as well-off as under $DA$. The proposition then follows from the fact that $i$ must be at least as well-of in the equilibrium $\sigma^*$ as she is when she plays $\ddot{\sigma}$ (and others play according to $\sigma^*$).

Let $\ddot{\sigma}_\ell := \sum_{\boldsymbol{v}\in\tilde{\mathcal{V}}} \sigma_\ell^*(\boldsymbol{v}) f(\boldsymbol{v})$. That is, $\ddot{\sigma}_\ell$ involves playing $\sigma_\ell^*(\boldsymbol{v})$ with probability $f(\boldsymbol{v})$, i.e., according to the probability distribution of priority-types that play that strategy in the equilibrium $\sigma^*$. For any school $s_a$, the probability that the student is assigned to $s_a$ when she plays $\ddot{\sigma}_\ell$ and others play the equilibrium strategy is

$$\dot{P}_a^\ell(\ddot{\sigma}_\ell) = \dot{P}_a^\ell\left(\sum_{\boldsymbol{v}\in\tilde{\mathcal{V}}} \sigma_\ell^*(\boldsymbol{v}) f(\boldsymbol{v})\right) = \sum_{\boldsymbol{v}\in\tilde{\mathcal{V}}} \dot{P}_a^\ell(\sigma_\ell^*(\boldsymbol{v})) f(\boldsymbol{v}) = \frac{q_a - h\sum_{\boldsymbol{v}\in\tilde{\mathcal{V}}} \dot{P}_a^h(\sigma_h^*) f(\boldsymbol{v})}{\ell}, \quad (3)$$

were the last equality follows from (2).

Observe that (3) implies

$$\dot{P}_y^\ell(\ddot{\sigma}_\ell) \leq \hat{P}_y^\ell, \qquad \text{for every low-quality school } s_y \in \{s_{g+1}, \ldots, s_m\}. \quad (4)$$

Because $\hat{P}_a^\ell = 0$ for all $a \in \{g+1, \ldots, m\}$, we also have

$$\dot{P}_x^\ell(\ddot{\sigma}_\ell) \geq \hat{P}_x^\ell, \qquad \text{for every high-quality school } s_x \in \{s_1, \ldots, s_g\}. \quad (5)$$

Specifically, any inequality in (4) is strict if $\sum_{\boldsymbol{v}\in\tilde{\mathcal{V}}} \dot{P}_y^h(\sigma_h^*) f(\boldsymbol{v}) > 0$. That is, one of these inequalities is strict if the equilibrium of $IA$ is blending. As a consequence, if the equilibrium of $IA$ is blending, *some* of the inequalities in (5) must also be strict.

Together, (4) and (5) imply that for any valuation-type $\boldsymbol{v} \in \bar{\mathcal{V}}$,

$$V_{\ell,\boldsymbol{v}}^{IA} \geq \sum_{a\in A} v_a \dot{P}_a^\ell(\sigma_\ell^*) \geq \sum_{a\in A} v_a \hat{P}_a^\ell = V_{\ell,\boldsymbol{v}}^{DA},$$

where the last inequality is strict if the equilibrium of $IA$ is blending. □

By definition of $V_p^X$, we have the following *ex-ante* corollary of Theorem 2.

**Corollary 4.** *For any priority-independent distribution $f$ and any symmetric equilibrium of $IA$, we have $V_\ell^{IA} \geq V_\ell^{DA}$, and $V_\ell^{IA} > V_\ell^{DA}$ if the equilibrium of $IA$ is blending.*

Abdulkadiroğlu et al. (2011) show that in the absence of pre-existing priorities (i.e., when, unlike in our model, *all* priorities are determined through tie-breaking), all students are always better-off under $IA$ than under $DA$ (as opposed to low priority-types only in Theorem 2). As Troyan (2012) shows, this is not true in the presence of pre-existing priorities. In particular, with two priority-types as in the present model, some high priority-types can be worse-off under $IA$ than under $DA$ (Troyan, 2012, Examples 1 and 2). This is intuitive: In $DA$, high priority-types are guaranteed an assignment at one of the high-quality schools whereas in $IA$, low priority-types can "steal" seats at high-quality schools, hence making some high priority-types worse-off than in $DA$.

Troyan (2012) however shows that from an ex-ante perspective, $IA$ remains preferable to $DA$ *even* in the presence of pre-existing priorities. That is, when one considers a student's expected utility *before* she draws her priority-type, the student is better-off under $IA$ than under $DA$. In our context and our terminology, Proposition 2 in Troyan (2012) notably implies that the allocative welfare of $IA$ is larger than the allocative welfare of $DA$, i.e, $V^{IA} \geq V^{DA}$.

Importantly, this result requires to assume that the distribution of priorities is the same for every student. In practice, this is rarely the case. Students' priority-types typically correlate with their characteristics such as parents' income, home location, intellectual abilities, etc. This puts in question the use of allocative efficiency as a social objective: If $V^X > V^Y$ but $Y$ favors disadvantaged students, one may have a legitimate preference for $Y$. The proof of theorem 2 show that $IA$ promotes access to high-quality schools for students with low priorities (as was already suggested in a special case by Abdulkadiroğlu et al., 2011, Theorem 3). Suppose that disadvantaged students tend to be the students with low priorities, as can be expected, for example, if priorities follow from test-scores. Then Theorem 2 indicates that $IA$ is better than $DA$ at helping disadvantaged students secure a higher-quality school, which complements and reinforces Troyan's comparison in terms of allocative welfare.[18]

Together with most of the school choice literature (including Abdulkadiroğlu et al., 2011), Troyan also focuses on the welfare effects of mechanism selection mediated by assignments to schools *themselves*. As we argued, the choice of a mechanism impacts welfare beyond determining students' assignments. In particular, by changing the value associated with different priority-types, a change in the allocation mechanism can change the "rent-seeking" behavior that lead to acquiring these priority-types. Mechanisms that increase the utility wedge between high and low priority-types may foster fiercer competition for higher priorities, which can increase wasteful effort. A natural questions is therefore whether the efficiency advantage of $IA$ over $DA$ is robust to the addition of an effort stage, i.e., whether net welfare $W(V_\ell^{IA}, V_h^{IA}, \boldsymbol{a})$ is also larger than net welfare $W(V_\ell^{DA}, V_h^{DA}, \boldsymbol{a}))$. The next proposition answers positively when valuations are priority-independent.

**Proposition 4.** *For any priority-independent distribution $f$ and any symmetric equilibrium of $IA$ (i) $W(V_\ell^{IA}, V_h^{IA}, \boldsymbol{a}) \geq W(V_\ell^{DA}, V_h^{DA}, \boldsymbol{a})$, and (ii) $W(V_\ell^{IA}, V_h^{IA}, \boldsymbol{a}) > W(V_\ell^{DA}, V_h^{DA}, \boldsymbol{a})$ if the equilibrium of $IA$ is blending and there is at least one student in the competitive set under $DA$ (i.e., $C(\Delta^{DA}, \boldsymbol{a}) \geq 1$).*

*Proof.* **(i).** By Corollary 4, $V_\ell^{IA} \geq V_\ell^{DA}$. Thus, if we also have $V_h^{IA} \geq V_h^{DA}$, the proposition follows directly from Proposition 1. Hence, assume that $V_h^{IA} < V_h^{DA}$. By Corollary 1, the difference between the two mechanisms' net welfare is

$$W(V_\ell^{IA}, V_h^{IA}, \boldsymbol{a}) - W(V_\ell^{DA}, V_h^{DA}, \boldsymbol{a}) = \underbrace{\left(V^{IA} - V^{DA}\right)}_{:=\Omega_1} + \underbrace{\left(D(\Delta^{DA}, \boldsymbol{a}) - D(\Delta^{IA}, \boldsymbol{a})\right)}_{:=\Omega_2}.$$

By Troyan (2012, Proposition 2), we have $V^{IA} \geq V^{DA}$, which implies $\Omega_1 \geq 0$. Also, together, $V_\ell^{IA} \geq V_\ell^{DA}$ and $V_h^{IA} < V_\ell^{DA}$ imply $\Delta^{DA} > \Delta^{IA}$. By definition of the deadweight

[18] An argument similar to the proof of Theorem 2 can be used to show that *regardless of the number of priority-types*, the *lowest* priority-type is always better-off under $IA$ than under $DA$. In that sense, the *most* disadvantaged students prefer $IA$ to $DA$ even when there are more than two priority-types.

loss term, this implies that the deadweight loss is larger in $DA$ than in $IA$, which in turn implies $\Omega_2 \geq 0$.

**(ii).** Because the equilibrium is blending we have $V_\ell^{IA} > V_\ell^{DA}$ by Corollary 4. Thus, if we also have $V_h^{IA} \geq V_h^{DA}$, the proposition follows directly from Proposition 1. Hence, assume that $V_h^{IA} < V_h^{DA}$ which together with $V_\ell^{IA} > V_\ell^{DA}$ implies $\Delta^{DA} > \Delta^{IA}$. Recall that $\Omega_1, \Omega_2 \geq 0$ by the proof of (i). Therefore, $\Omega_2 > 0$ is sufficient to have the desired result. By definition, the set of competitive students grows with $\Delta$, and the set of competitive students under $IA$ is therefore a subset of the same set under $DA$. Because only competitive students contribute to the deadweight loss and each of these students' contribution increases with $\Delta$, we therefore have $\Omega_2 > 0$. □

Importantly, the ex-ante efficiency advantage of $IA$ over $DA$ demonstrated by Troyan (2012, Proposition 2) relies on students having heterogeneous *cardinal* preferences over schools. When heterogeneities in cardinal preferences vanish, most mechanisms tend to provide the same allocative welfare, and it becomes impossible for one mechanism to dominate another in terms of ex-ante (allocative) efficiency.

**Observation 1.** *If $v_i = v$ for all $i \in \{1, \ldots, m\}$, then for any two non-wasteful mechanisms $M$ and $M'$, the allocative welfare $V^M = V^{M'}$. In particular, in this case, $V^{IA} = V^{DA}$.*

However, even in this extreme case (fully homogeneous cardinal preferences), Proposition 4 shows that $IA$ remains more efficient than $DA$ in terms of net welfare. This is because, even when no allocative efficiency gains are possible, $IA$ still favors low priority-types over high priority-types, which in turns means that wasteful competition is reduced in the effort stage.[19]

**Observation 2.** *Proposition 4 applies even when $v_i = v$ for all $i \in \{1, \ldots, m\}$ and $V^{IA} = V^{DA}$. In particular, in this case too, $W(V_\ell^{IA}, V_h^{IA}, \boldsymbol{a}) > W(V_\ell^{DA}, V_h^{DA}, \boldsymbol{a})$ if the equilibrium of $IA$ is blending and there is at least one student in the competitive set under $DA$.*

## 4.2 Priority-dependent valuations

In the presence of pre-existing priorities, the assumption that valuations are independent of priorities can be problematic. Instead, students' priority-types are often correlated with their valuation-types. For example, students who are living inside a neighborhood usually get higher priorities at the schools in that neighborhood. These students may also have a more intense preference for the schools in that neighborhood — because of shorter commute time or because parents want their children to attend the same school as their neighbors — which results in priority-dependent valuations.

Results in the previous section rely on priority-independent valuations. This is true, for example, of Proposition 4 which relies on Theorem 2 to show that the ex-ante efficiency advantage of $IA$ over $DA$ (Troyan, 2012, Proposition 2) is robust to the addition of an effort stage. When correlations are introduced, Theorem 2 does not necessarily apply and the picture becomes more complex.

[19] Formally, even when $V^{IA} = V^{DA}$, we still have $V_\ell^{DA} \leq V_\ell^{IA}$ and $V_h^{DA} \geq V_h^{IA}$, which by Corollary 4 implies $W(V_\ell^{IA}, V_h^{IA}, \boldsymbol{a}) \geq W(V_\ell^{DA}, V_h^{DA}, \boldsymbol{a})$

On the one hand, $IA$ enables efficiency gains by incentivizing students to reveal information about their cardinal preferences through their ordinal reports. Even when valuations correlate with priority-types, priorities that result from tie-breaking carry no information on cardinal utility. In an equilibrium of $IA$, it is possible for some of these priorities to be violated in a welfare-improving way (compared to $DA$).

On the other hand, $IA$ also enables violations of *pre-existing* priorities which carry useful information on cardinal utility if valuations are priority-dependent.[20] As a consequence, $IA$ may not take full advantage of correlations between priorities and valuations. In contrast, $DA$ respects priorities and therefore takes full advantage of correlations between preferences and priorities. $DA$ however does not incentivize cardinal preference revelation and therefore misses on some efficiency improvement opportunities.

To shed light on these two effects, we generalize the model of the previous section. We now assume that low priority-types and high priority-types draw valuation-types from different distribution $f_\ell$ and $f_h$, with $f_\ell \neq f_h$. Recall that the priority-types on which these distributions depend are *acquired* through the first effort stage. That is, students here develop a particular taste for some schools as *a result of* the effort they exerted to secure a higher priority at these schools.

We also relax the common ordinal preference assumption and let the finite set of possible utility values be $\bar{\mathcal{V}} \subset [0,1]^m$, without requiring that $v_1 > \cdots > v_m$ for every $\boldsymbol{v} \in \bar{\mathcal{V}}$. That is, although we keep the distinction between high- and low-quality schools in terminology, we allow students to prefer low-quality schools to high-quality schools (e.g., because they live closer to a low-quality school).

Allowing valuations to be correlated with priority-types, it is not hard to find examples of $f_\ell$ and $f_h$ for which $DA$ is more *allocatively* efficient than $IA$. For example, suppose that $f_\ell$ and $f_h$ satisfy the following properties:

a) For all $\boldsymbol{v}, \boldsymbol{v'}$ in the support of $(f_h)$, $v_i = v'_i$ for all $i \in \{1, \ldots, g\}$, and for all $\boldsymbol{v}, \boldsymbol{v'}$ in the support of $(f_\ell)$, $v_i = v'_i$ for all $i \in \{g+1, \ldots, m\}$.

b) If $\boldsymbol{v}$ is in the support of $(f_\ell)$ or $(f_h)$, then $v_i > v_j$ for all $i \in \{1, \ldots, g\}$ and all $j \in \{g+1, \ldots, m\}$.

c) For all $\boldsymbol{v}$ in the support of $f_h$ and all $\boldsymbol{v'}$ in the support of $f_\ell$, we have $v_i > v'_i$ for all $i \in \{1, \ldots, g\}$

In words, property a) says that cardinal preferences are homogeneous at high-quality schools among high priority-types, and at low-quality schools among low priority-types. Property b) says that ordinal preferences between high- and low-quality schools are maintained: Every student still prefers any high-quality school to any low-quality school. Finally, property c) says that the value a high priority-type associates to a high-quality school is alway higher than the value a low priority-type associates to that school. If $f_\ell$ and $f_h$ satisfy a), b), and c), we say that $(f_\ell, f_h)$ **favors DA**.

If $(f_\ell, f_h)$ favors DA, then $DA$ is more allocatively efficient than $IA$. Intuitively, a) cuts the opportunities for efficiency improvements in $IA$, while c) makes respecting

[20] In $IA$, a student from a given district may be assigned with positive probability to a school in another district, even if she has a lower average value at that school than the average value among students in the district of that school.

priorities optimal from the point of view of allocative efficiency. Finally b) guarantees that the (dominant strategy) outcome of $DA$ respects priorities.

**Observation 3.** *If* $(f_\ell, f_h)$ *favors DA, then* $V^{DA} \geq V^{IA}$ *and* $V^{DA} > V^{IA}$ *if the equilibrium of* $IA$ *is blending.*

*Proof.* By c), allocating a high-quality school to a high priority-type is always preferable to allocating the same school to a low priority-type from the point of view of allocative efficiency. By a), given that high-quality schools are allocated to high priority-types only, the allocation of these schools among high priority-types is irrelevant from the point of view of allocative efficiency. Similarly, by a), if low-quality schools are allocated to low priority-types only, then the allocation of these schools among low priority-types is irrelevant from the point of view of allocative efficiency.

Thus, any allocation of the schools that assigns high priority-types to high-quality schools and low priority-types to low-quality schools exclusively is optimal from the point of view of allocative efficiency. By b), this is true of every allocation in the support of $DA$ given $f_\ell$ and $f_h$. Hence $V^{DA}$ is optimal and we have $V^{DA} \geq V^{IA}$. It is also straightforward from the above argument that $V^{DA} > V^{IA}$ if the equilibrium of $IA$ is blending. □

More generally, $V^{DA} > V^{IA}$ if $DA$'s efficiency gains from exploiting correlations between priorities and valuations outweigh $IA$'s efficiency gains from incentivizing cardinal preference revelation, which may occur under conditions on $f_\ell$ and $f_h$ milder than $DA$-favorability.

Interestingly, for a class of pairs $(f_\ell, f_h)$ including DA-favorable pairs, Theorem 2 extends to the more general model studied in this section. That is, for $(f_\ell, f_h)$ in this class, the low priority-types remain better-off in $IA$ than in $DA$. In turn, this implies that even if $V^{DA} > V^{IA}$, the net welfare of $DA$ remains lower than the net welfare of $IA$ when $\boldsymbol{a}$ is sufficiently competitive.

Consider the following property of $f_\ell$:

d) For all $\boldsymbol{v}$ in the support of $f_\ell$, we have $v_{g+1} > v_{g+2} > \cdots > v_m$.

In words, property d) says that *ordinal* preferences over low-quality schools are homogeneous *among low priority-types*. If $(f_\ell, f_h)$ satisfies b) and d), we say that $(f_\ell, f_h)$ is **sufficiently homogeneous**. Clearly, a) implies d) and $(f_\ell, f_h)$ being DA-favorable implies that the pair is also sufficiently homogeneous.

The following theorem generalizes Theorem 1 to priority-dependent valuations under the assumption that $(f_\ell, f_h)$ is sufficiently homogeneous.

**Theorem 3.** *For any sufficiently homogeneous* $(f_\ell, f_h)$*, any valuation-type* $\boldsymbol{v} \in \bar{\mathcal{V}}$*, and any symmetric equilibrium of* $IA$*, we have* $V^{IA}_{\ell,\boldsymbol{v}} \geq V^{DA}_{\ell,\boldsymbol{v}}$*, and* $V^{IA}_{\ell,\boldsymbol{v}} > V^{DA}_{\ell,\boldsymbol{v}}$ *if the equilibrium of* $IA$ *is blending.*

*Proof.* By b), high priority-types always prefer high-quality schools to low-quality schools. Hence, high priority-types always rank all high-quality schools higher than any low-quality school in the dominant strategy equilibrium of $DA$. In $DA$, all the seats at high-quality schools are therefore assigned uniformly at random to high priority-types. By d), the low priority-type all report the same ranking over low-quality schools. Because all seats at high-quality schools are occupied by high priority-types, the $\ell$ low priority-type are therefore assigned uniformly at random to one of the $\ell$ low-quality schools. Therefore, the probability that a low priority-type student is assigned to some school $s_a$ under the dominant strategy outcome of $DA$ is again $\hat{P}_a^\ell$.

The rest of the proof is similar to the proof of Theorem 2. For every school $s_a$, the feasibility constraint (respecting quotas at schools) and the fact that $IA$ is non-wasteful now imply that at any equilibrium $\sigma^*$ of $IA$,

$$\ell \sum_{\boldsymbol{v}\in\bar{\mathcal{V}}} \dot{P}_a^\ell(\sigma_\ell^*) f_\ell(\boldsymbol{v}) + h \sum_{\boldsymbol{v}\in\bar{\mathcal{V}}} \dot{P}_a^h(\sigma_h^*) f_h(\boldsymbol{v}) = q_a, \tag{6}$$

where compared to (2), we have only added indices $\ell$ and $h$ to the density functions.

The strategy that makes any type in $IA$ at least as well-off as in $DA$ is now $\ddot{\sigma}_\ell := \sum_{\boldsymbol{v}\in\bar{\mathcal{V}}} \sigma_\ell^*(\boldsymbol{v}) f_\ell(\boldsymbol{v})$, where again, we only added the superscript "$\ell$" to the distribution (compared to the corresponding strategy in the proof of Theorem 2).

We then have

$$\dot{P}_a^\ell(\tilde{\sigma}_\ell) = \dot{P}_a^\ell\left(\sum_{\boldsymbol{v}\in\bar{\mathcal{V}}} \sigma_\ell^*(\boldsymbol{v}) f_\ell(\boldsymbol{v})\right) = \sum_{\boldsymbol{v}\in\bar{\mathcal{V}}} \dot{P}_a^\ell(\sigma_\ell^*(\boldsymbol{v})) f_\ell(\boldsymbol{v}) = \frac{q_a - h\sum_{\boldsymbol{v}\in\bar{\mathcal{V}}} \dot{P}_s^h(\sigma^*) f_h(\boldsymbol{v})}{\ell}, \tag{7}$$

where the last equality follows from (6). Again, (7) implies (4) and (5), with each individual inequality in (4) being strict if $\sum_{\boldsymbol{v}\in\bar{\mathcal{V}}} \dot{P}_y^h(\sigma^*) f_h(\boldsymbol{v}) > 0$. Together, (4), (5), and b) imply that for any valuation-type $\boldsymbol{v} \in \mathcal{V}$,

$$V_{\ell,\boldsymbol{v}}^{IA} \geq \sum_{a\in A} v_a \dot{P}_a^\ell(\sigma_\ell^*) \geq \sum_{a\in A} v_a \hat{P}_a^\ell = V_{\ell,\boldsymbol{v}}^{DA},$$

where the last inequality is strict if the equilibrium of $IA$ is blending. □

Clearly, by definition of $V_p^X$, the following is a direct corollary of Theorem 3, and generalizes Corollary 4.

**Corollary 5.** *For any sufficiently homogeneous $(f_\ell, f_h)$ and any symmetric equilibrium of $IA$, we have $V_\ell^{IA} \geq V_\ell^{DA}$, and $V_\ell^{IA} > V_\ell^{DA}$ if the equilibrium of $IA$ is blending.*

Corollary 5 implies that although net welfare may be higher under $DA$ than under $IA$ when valuations are priority-dependent, this can only happen if the allocative welfare of $DA$ is higher than the allocative welfare of $IA$. *Even* when $DA$ is allocatively more efficient than $IA$, the net welfare also remain higher under $IA$ than under $DA$ if the effort stage is sufficiently competitive.

**Proposition 5.** *Suppose that $(f_\ell, f_h)$ is sufficiently homogeneous. (i) If $V^{IA} \geq V^{DA}$, then $W(V_\ell^{IA}, V_h^{IA}, \boldsymbol{a}) \geq W(V_\ell^{DA}, V_h^{DA}, \boldsymbol{a})$. (ii) There exists $\boldsymbol{a}$ sufficiently competitive such that $W(V_\ell^{IA}, V_h^{IA}, \boldsymbol{a}) \geq W(V_\ell^{DA}, V_h^{DA}, \boldsymbol{a})$ even if $V^{DA} > V^{IA}$.*

*Proof.* **(i).** We have $V^{IA} \geq V^{DA}$ and $V_\ell^{IA} \geq V_\ell^{DA}$ and the proof is therefore identical to the proof of Proposition 4(i). **(ii).** The proof follows directly from Corrolary 5 and Proposition 2.

□

# 5 Conclusion

In this paper, we find that, in a context where students compete for priority in a school choice mechanism, $IA$ may have better welfare properties than $DA$. Troyan (2012) and Abdulkadiroğlu et al. (2011) also provide cases in which $IA$ provides higher welfare than $DA$ but purely from an allocation perspective with exogenous priorities. Our results extend this qualitative conclusion by showing that $IA$ can be preferable to $DA$ from a *net* welfare perspective including the cost for students to compete for priorities. As we show, this is true *even* when $DA$ provides a more efficient allocation than $IA$.

We stress, however, that our results should not been seen an unconditional defense of $IA$. In particular, we have shown in Section 4.2 that when valuations in the matching stage depend on the priorities acquired through the effort stage, $DA$ can have better allocative efficiency than $IA$. For $IA$'s *overall* efficiency to dominate $DA$'s, the effort stage then needs to be sufficiently competitive so that $DA$'s larger inequalities (between high and low priority-types) induce a large wasteful effort with $DA$ that overwhelm its advantage in terms of allocative efficiency.

Overall, our main goal was *not* to provide general conclusions on the relative efficiency of $IA$ and $DA$. Rather, we have attempted to draw attention to the interactions between the choice of a mechanism and the game through which participants acquire priorities. More generally, we have shown how the choice of a mechanism can have economically significant impacts beyond the problem the mechanism is specifically designed to solve.

To further illustrate how the balance can "tip back" in favor of $DA$, we conclude by discussing another situation in which interactions between the school choice mechanism and the effort stage may results in $DA$ being more efficient than $IA$.

Throughout this paper, we have assumed students do *not* know the realization of their utility value for schools at the time they exert effort in order to secure higher priorities. Instead, we have assumed that prior to the matching stage, students only know the *distribution* from which they will draw values for schools in the matching stage but do *not* know *the realization* of this distribution.[21]

Suppose instead that students have intrinsically heterogeneous utility over schools, and these utilities are known at the beginning of the effort stage. Students can then signal their heterogeneous valuation directly through the effort stage. This can potentially reverse the efficiency comparison between $DA$ and $IA$. To understand why, notice that if students with a more intense preference for higher-quality schools exert more effort and obtain higher priorities as a consequence, priorities become a signal of preference intensity. In this case, respecting priorities may become desirable from the point of view of efficiency. Because $DA$ is better at respecting priorities than $IA$, $DA$ could therefore be more efficient than $IA$.[22]

[21] Signaling valuation through effort is discussed to a certain extent in Section 4.2. In Section 4.2, however, students do not differ in their potential valuations over schools *at the beginning of the effort stage*. Students rather *acquire* different valuations over schools as they gain different priorities over these schools (e.g., students acquire a higher preference for schools in a given neighborhood by moving to that neighborhood, which also gives them a higher priority at the schools in that neighborhood). Here, in comparison, we discuss a variant of our model where students are *initially* heterogeneous in their valuation over schools (i.e., heterogeneous from the very beginning of the effort stage) and are able to signal their heterogeneous valuation through heterogeneous efforts.

[22] Although this is different from the kind of signaling discussed in Section 4.2 (see footnote 21), the

An example where signaling of intrinsic valuation through effort makes $DA$ more efficient than $IA$ is presented in the Appendix. As the example illustrates, these efficiency gains can materialize *even when effort-types are perfectly competitive* with $a_i = a$ for all $i \in \{1, \ldots, n\}$, which contrasts with Corollary 5. Again, this example points at the importance of better understanding the interaction between the choice of mechanism and the game through which students acquire priorities at schools. More generally, it stresses the value of considering mechanism selection in "general equilibrium" settings, rather than focusing exclusively on the partial equilibrium effect a mechanism has on the allocation problem it is designed to solve.

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intuition is similar to that presented in Section 4.2.

# A Appendix

## A.1 Stage 1 Contest is *Generic*

Let $\tilde{u}_i(t) = u_i(t) - V_\ell$. Rewrite $\tilde{u}_i(t) = H_i(t)\, v_i(t_i) - (1 - H_i(t))\, c_i(t_i)$ where $v_i(t_i)$ and $c_i(t_i)$ are defined follows:

$$v_i(t_i) := V_h - V_\ell - (e(t_i) - e(a_i))$$

$$c_i(t_i) := (e(t_i) - e(a_i))$$

$\tilde{u}_i(t)$ is a normalization of $u_i$ with the same constant $V_\ell$ subtracted from each players utility. With this normalization, the contest meets $A1 - A3$ and the *power* and *cost* conditions of Siegel (2009).

Since $e$ is assumed to be continuous, $v_i$ and $-c_i$ are continuous. Since $e$ is assumed to be strictly increasing, $v_i$ and $-c_i$ are strictly-decreasing ($A1$). $v_i(a_i) = V_h - V_\ell > 0$ by assumption ($A2$). Since $v_i(t_i) := V_h - V_\ell - c_i(t_i)$, when $v_i(t_i) = 0 \Rightarrow c_i(t_i) = V_h - V_{\ell;} > 0$ ($A3$). Let the *reach* of a player $r_i$, be the score such that $v_i(r_i) = 0$. This can also be written $e(r_i) = V_h - V_\ell + e(a_i)$. Since the effort function $e$ is identical for each player and is assumed to be strictly increasing, and since the players are indexed by $a_i$, $r_i \geq r_j$ for $i < j$. The *threshold* $\tilde{t} = r_{h+1}$. A players *power* is $v_i(\tilde{t}) = V_h - V_\ell - \left(e(\tilde{t}) - e(a_i)\right)$. Note that $v_{h+1}(\tilde{t}) = v_{h+1}(r_{h+1}) = 0$. Since, by assumption, $a_{h+1} \neq a_j$ for $j \neq h+1$, $v_i(\tilde{t}) \neq 0$ for all $i \neq h+1$ (*Power Condition*). Since $v_i(t_i)$ is strictly decreasing for all players, it is strictly decreasing for $h+1$ at $\tilde{t}$ (*Cost Condition*).

## A.2 Signaling differences in intrinsic valuation through effort

Consider a variation of our model where students have different valuation-types $f_i(\boldsymbol{v})$. That is, students now draw valuations over schools from idiosyncratic distributions *which they know at the beginning of the effort stage.* This is unlike the model presented in the paper where *all* high or low priority-types draw a valuation from the same distribution $f_\ell$ or $f_h$, respectively. In that case, valuation-types are "acquired" through priorities. Here, we assume that valuation-types are independent of priority-types, and may differ from one student to another. In particular, unlike in Section 4.2, it is in principle possible for a high valuation-type to have a low priority-type, and vice-versa.

In this case, we want to show that $DA$ can be more efficient than $IA$ even in a perfectly competitive environment, and even when $DA$ induces more wasteful effort. The reason is, in this new setting, effort becomes a *signal* of students heterogeneous valuations $f_\ell$ or $f_h$. Therefore, even if effort is costly and $DA$ pushes students to exert more effort than $IA$, this can be more than compensated by the usefulness of the signal from an efficiency standpoint.

To rule out efficiency results driven by insufficient competitiveness, we assume $a_i = a$ for all $i \in \{1, \ldots, n\}$. For simplicity, we also assume a *linear* cost of effort. There are four students and three schools, $s_{H_1}$, $s_{H_2}$, and $s_L$. Schools $s_{H_1}$ and $s_{H_2}$ have one seat, and school $s_L$ having two seats. We consider the case of two valuation-types $h$ and $\ell$ with degenerate valuation distributions $f_h$ and $f_\ell$ with $f_h(\boldsymbol{v}^h) = 1$ and $f_\ell(\boldsymbol{v}^\ell) = 1$, where $\boldsymbol{v}^h$ and $\boldsymbol{v}^\ell$ are defined as follows:

| Schools | $H_1$ | $H_2$ | $L$ |
|---|---|---|---|
| $\boldsymbol{v}^h$ | .8 | .2 | 0 |
| $\boldsymbol{v}^\ell$ | .55 | .25 | .20 |

There are two high valuation-types who draw values from $f_h$, and two low-valuation-types who draw values from $f_\ell$. Students still compete for a high priority at the high-quality schools $H$ and $HM$ through effort, with the two highest effort students getting high priorities at the high-quality schools (ties are again broken symmetrically at random).

Let $V^X_{i,j}$ denote the value for allocative welfare of valuation-type $i$ when being priority-type $j$ when the mechanism is $X$. For example, $V^{IA}_{h,\ell}$ is the value for a high valuation-type of being of being priority-type $j$ when the mechanism is $IA$ We now compute valuations depending on whether types are aligned, with high valuation-types having high priority-types, or misaligned, with high valuation-types having low priority-types.

$IA$ **with aligned types.** Observe that ranking school $s_L$ anywhere but last is dominated for all students. Thus, students must choose between strategy $Q_1$ that consists in ranking $s_{H1}$ first followed by $s_{H2}$, and strategy $Q_2$ that consists in ranking $s_{H2}$ first followed by $s_{H1}$ (or a mix of these two strategies).

For high valuation-types, $Q_1$ is a dominant strategy.[23] The best response for low valuation-types is therefore $Q_2$, which yields the following valuations:

- $V^{IA}_{h,h} = (1/2) * 0.8 + (1/2) * 0 = 0.4$, and
- $V^{IA}_{\ell,\ell} = (1/2) * 0.25 + (1/2) * 0.20 = 0.225$.

$IA$ **with misaligned types.** Again, ranking school $s_L$ anywhere but last is dominated for all students, and students strategies $Q_1$ and $Q_2$ (or a mix thereof). For low valuation-types, $Q_1$ is a dominant strategy.[24] The best response for low valuation-types is therefore $Q_2$, which yields the following valuations:

- $V^{IA}_{h,\ell} = (1/2) * 0.2 + (1/2) * 0 = 0.1$, and
- $V^{IA}_{\ell,h} = (1/2) * 0.55 + (1/2) * 0.20 = 0.375$.

$DA$ **with aligned types.** Valuations are:

- $V^{DA}_{h,h} = (1/2) * 0.8 + (1/2) * 0.2 = 0.5$, and
- $V^{DA}_{\ell,\ell} = 0.20$.

$DA$ **with misaligned types.** Valuations are:

[23] Strategy $Q_1$ is a strict best-response both when the other high valuation-type reports $Q_1$ and when the other high valuation-type reports $Q_2$.

[24] Again, strategy $Q_1$ is a strict best-response both when the other low valuation-type reports $Q_1$ (yielding 0.3 as a payoff, instead of 0.25 under $Q_2$) and when the other low valuation-type reports $Q_2$.

- $V^{DA}_{h,\ell} = 0$, and
- $V^{DA}_{\ell,h} = (1/2) * 0.6 + (1/2) * 0.20 = 0.4$.

This implies

$$V^{IA}_{h,h} - V^{IA}_{h,\ell} = 0.3 > 0.175 = V^{IA}_{\ell,h} - V^{IA}_{\ell,\ell},$$

and

$$V^{DA}_{h,h} - V^{DA}_{h,\ell} = 0.5 > 0.2 = V^{DA}_{\ell,h} - V^{DA}_{\ell,\ell}.$$

Thus, in an equilibrium of the effort stage with linear effort functions, the high valuation-types exert the most effort and become high priority-types. Specifically, high valuation-types exert 0.175 effort when the mechanism is $IA$ and 0.275 when the mechanism is $DA$ (low valuation-types exert no effort).

Therefore the net welfare under each mechanism is:

- $W(V^{IA}_{\ell}, V^{IA}_{h}, \boldsymbol{a}) = 2 * [0.4 - 0.175] + 2 * 0.225 = 0.9$, and
- $W(V^{DA}_{\ell}, V^{DA}_{h}, \boldsymbol{a}) = 2 * [0.5 - 0.2] + 2 * 0.2 = 1$.

That is, the higher effort cost inherent with $DA$ is more than compensated by the fact that $DA$ makes better use of the valuation-type signaling contained in that effort. Of course, this is just one example and other valuation profiles can be found for which the additional cost of effort in $DA$ outweighs $DA$'s better use of effort signaling. This again points to the importance of better understanding the mechanisms through which students acquire priorities and their interactions with allocation mechanisms, and we leave potential general results in the case of intrinsic valuation heterogeneities for future research.

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