THE COORDINATED VOLUNTEER'S DILEMMA†

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ABSTRACT. In the Volunteer's Dilemma, the effort of one player can benefit an entire group, but all who volunteer pay a cost. In a well-known result, Diekmann (1985) demonstrates that in equilibrium, larger groups are *less likely* to find even one volunteer. In this paper, we show that, remarkably, even if effort is coordinated and only one of the volunteers is randomly chosen to bear the cost, larger groups are still less likely to produce a volunteer.

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> "... every one is more negligent of what another is to see to, as well as himself, than of his own private business; as in a family one is often worse served by many servants than by a few." *Aristotle's Politics* Book II, Chapter 3

1. Introduction: The Volunteer's Dilemma

For many tasks, the efforts of a single individual are sufficient to serve an entire group. One sentinel can alert the entire community. One group member can investigate and report on the honesty of a vendor or the usefulness of a consumer good. Thus we might expect clustering into groups to be advantageous.

But, as the epigraph from Aristotle suggests, there is a countervailing force. The returns to scale offered by larger groups can be offset by free-rider problems that increase with group size. In this passage, Aristotle makes two assertions, the first claim is that as group size increases, each member is less likely to act in the public interest. The second is the stronger claim that *total* effort diminishes with group size.

The forces that underlie Aristotle's claims are nicely captured in a simple n-player game that was introduced by the sociologist, Andreas Diekmann, who named this game, the *Volunteer's Dilemma* (Diekmann, 1985). In Diekmann's Volunteer's Dilemma game, as originally presented, players have identical payoff functions and move simultaneously. Each player has the option of taking a costly action *volunteer* or of choosing *not volunteer*. If at least one player chooses *volunteer*, then all players receive benefits b. Those who choose not to volunteer have net benefits b, while those who choose to volunteer have net benefits $b-c>0$. If no player volunteers, then all get a payoff of 0.

In this game, with two or more players, there can not be a Nash equilibrium where everyone volunteers. If everyone else volunteers, a player is better off not volunteering than volunteering. Nor can there be an equilibrium where no one volunteers. If no one else volunteers, a player is better off volunteering than not.

Diekmann shows that for a group of size n, there is a unique symmetric mixed-strategy Nash equilibrium with the following properties.

Proposition 1 (Diekmann)**.** In a symmetric Nash equilibrium for the Volunteer's Dilemma, as the number of players increases:

1.1 The probability that each player volunteers diminishes.
1.2 The probability that no player volunteers increases and asymptotically approaches $\frac{c}{b}$ from below.
1.3 The expected utility of each player remains constant and equal to $b-c$.

Proof. Any pure strategy equilibrium must be mixed. In a pure-strategy symmetric Nash equilibrium, either all players volunteer in which the best response is to not volunteer, or no player volunteers in which case the best response is to volunteer.

In a symmetric equilibrium, let q be the probability that a player does not volunteer. For any player, the probability that somebody else volunteers is then $1-q^{n-1}$. A player will be indifferent between volunteering and not volunteering if

$$b\left(1-q^{n-1}\right)=b-c. \tag{1}$$

Equation 1 implies that the probability that any single player does not volunteer is

$$q=\left(\frac{c}{b}\right)^{\frac{1}{n-1}}. \tag{2}$$

Therefore, the probability that nobody volunteers is

$$q^n=\left(\frac{c}{b}\right)^{\frac{n}{n-1}}. \tag{3}$$

Result 1.1 follows from Equation 2 and the fact that $\frac{c}{b} < 1$. The probability of not volunteering is increasing in n and thus, the probability of volunteering is decreasing in n.

Results 1.2 follow from Equation 3 and the fact that $\frac{c}{b} < 1$. The probability of no one volunteering (q^n) is increasing in n and thus and approaches $\frac{c}{b}$ as n approaches infinity.

Result 1.3 follows from the fact that in equilibrium, players must be indifferent between, volunteering, not volunteering, and thus any mixture of these two. Since the value of volunteering is always $b-c$, the value of the equilibrium mixed strategy must also be $b-c$.

□

Result 1.1, that q is increasing in n, echoes Aristotle's assertion that "everyone is more negligent of what another is to see to, as well as himself...." *Result 1.2*, that q^n is increasing in n, affirms Aristotle's stronger claim that "one is often worse served by many servants than a few." Aristotle's proclamations concerned the well-being of the served, but not that of the servants. *Result 1.3* relates to the well-being of the servants. In Diekmann's model, although servants would like to see the job done, they are no worse off as their number increases because each of them is less likely to have to do the job themselves.

In the Volunteer's Dilemma, everyone who offers to volunteer must bear the cost of volunteering, even though only one player's volunteer is needed. Sometimes the efforts of volunteers can be managed more efficiently. An organization may be able to solicit volunteers for a one-person task and then choose just one volunteer who is asked to perform the task.

For example, the *National Marrow Donor Program* (NMDP) maintains a registry of persons who avow their willingness to donate stem cells to a leukemia patient with a matching immune system if the need should occur. For patients with common immunity types, there are likely to be many eligible volunteers in the registry. The NMDP *chooses just one* of the matching registrants to make the donation (Bergstrom et al., 2009).

2. The Coordinated Volunteer's Dilemma

2.1. *Game and Equilibrium*

In an n-player symmetric *Coordinated Volunteer's Dilemma*, there is a task to be performed. Each player has two possible pure strategies: *Volunteer* and *Not Volunteer*. If no player volunteers, the task is not done and all players receive a payoff of 0. If one or more players volunteer, exactly one volunteer is *randomly selected* to perform the task. In this case, each player i receives a benefit b. If player j is selected to do the task, then player j must pay a cost c and thus receives a net benefit of $b-c$.

In a symmetric Nash equilibrium where each player chooses *Volunteer* with probability $p = 1-q$ where $0 < q < 1$, it must be that the expected payoff for this mixed strategy is the same as that for the pure strategy *Not Volunteer*.

If each of the n players volunteers with probability $1-q$, then there will be at least one volunteer with probability $1-q^n$. Thus the expected benefit for each player is $b(1-q^n)$. If there is at least one volunteer, that volunteer is equally likely to be any of the n players. Thus the expected cost to each player is $\frac{1}{n}c(1-q^n)$. Therefore if each player volunteers with probability q, the expected payoff to each of them is

$$b(1-q^n) - \frac{c(1-q^n)}{n} \tag{4}$$

If all other players volunteer with probability $1-q$, then the expected payoff to *not Volunteer* is $b\left(1-q^{n-1}\right)$.

Therefore in a symmetric Nash equilibrium it must be that

$$b(1-q^n) - \frac{c(1-q^n)}{n} = b\left(1-q^{n-1}\right) \tag{5}$$

Rearranging terms of Equation 5, we see that this equation is equivalent to:

$$n\left(\frac{q^n}{1-q^n}\right)\left(\frac{1-q}{q}\right)=\frac{c}{b} \tag{6}$$

Define $\rho=\frac{c}{b}$ and $H(q,n)\equiv n\left(\frac{q^n}{1-q^n}\right)\left(\frac{1-q}{q}\right)$. We now state the equilibrium in the form of a lemma for use in the proof of our main proposition. *Proposition 2.1.*

Lemma 1. In a symmetric Nash equilibrium,

$$H(q,n)=\rho \tag{7}$$

One additional lemma regarding the shape of $H(q,n)$ is useful in the proof of our main result. The proof appears in the appendix.

Lemma 2. The function $H(q,n)$ has the following properties:

- $H(q,n)$ is continuous and strictly increasing in q.
- $H(0,n)=0$ and $\lim_{q\to 1} H(q,n)=1$
- $H(q,n)$ is strictly decreasing in n.

2.2. *Main Result*

We now present our results in an analogous way to Proposition 1. Below, W_{-1} represents the minor branch of Lambert W function.[1]

For a review of the properties of Lambert W function

Proposition 2 (Coordinated)**.** In the unique symmetric Nash equilibrium for the Coordinated Volunteer's Dilemma, as the number of players increases:

2.1 The probability that each player volunteers diminishes.
2.2 The probability that no player volunteers increases and asymptotically approaches $-\frac{\rho}{W_{-1}(-\rho e^{-\rho})}$.
2.3 The expected utility of each player is increasing.

Result 2.1, informs us that Aristotle's remark "every one is more negligent of what another is to see to." applies to coordinated as well as uncoordinated Volunteer's Dilemmas. It is not surprising that as the number of players increases, the probability that any one of them volunteers diminishes.

However, surprisingly, *even with coordination of volunteers*, *Result 2.2* tells us that the symmetric equilibrium probability that nobody volunteers increases with the number of players. Thus Aristotle's dictum that "one is often worse served by many servants than by a few" applies in strong form, with the word "often" replaced by "always".

Together, these results show that coordination does not solve the tension observed by Aristotle. From the standpoint of the served, larger groups are less effective. However, in contrast to the standard volunteer's dilemma, *Result 2.3* demonstrates that the servants are *better off* as their number increases.

With these lemmas in hand, we can prove the four results in Proposition 2.1.

2.3. *Proof q is Increasing in n.*

Proof. Lemma 2 informs us that $H(q,n)$ is a continuous increasing function of q on the interval $[0,1)$ and that $H(0)=0$ and $\lim_{q\to 1}=1$. It follows that for any $\rho\in[0,1)$ and any integer $n\geq 2$, the equation $H(q,n)=\rho$ has a unique solution $q(\rho,n)$.

Suppose that $n'>n$. By definition of the function $q(\rho,n')$, it must be that $H(q(\rho,n'),n')=\rho=H(q(\rho,n),n)$. According to Lemma 2, $H(q,n)$ is a decreasing function of n. Therefore $H\left(q(\rho,n),n'\right)<$

[1]Lambert W function is the multi-valued inverse of the function $f(x)=xe^x$. Properties and a review of the Lambert W can be found in Corless et al. (1996).

$H\left(q(\rho,n)\right)=\rho$. Since $H(q,n)$ is an increasing function of q, it must be that $q(\rho,n')>q(\rho,n)$. Therefore $q(\rho,n)$ is an increasing function of n.[2] □

2.4. *Proof that Expected Utility is Increasing in n.*

Proof of Result 2.3. In equilibrium, the expected utility is equal to the value of not volunteering which is $b\left(1-q\left(\rho,n\right)^{n-1}\right)$. Thus, expected utility is increasing in n if and only if

$$\frac{\partial\left(b\left(1-q\left(\rho,n\right)^{n-1}\right)\right)}{\partial n}>0 \tag{8}$$

This is equivalent to:

$$-bq^{n-1}\ln\left(q\left(\rho,n\right)\right)\frac{\partial\left(q\left(\rho,n\right)\right)}{\partial n}>0 \tag{9}$$

This is true since $b>0, q^{n-1}>0, \ln\left(q\left(\rho,n\right)\right)<0$ and $\frac{\partial(q(\rho,n))}{\partial n}>0$ (by *Result C*). □

2.5. *Proof Q is Increasing in n.*

The proof of Result *2.3* relies on two additional lemmas which are proved in the appendix.

Lemma 3. If $0<x<1$ and n is a positive integer, then

$$\frac{x(1-x^n)}{1-x^{n+1}}<\frac{n}{n+1} \tag{10}$$

Lemma 4. For all $q\in(0,1)$ and all integers $n\geq 1$, $H\left(q^{n/n+1},n+1\right)<H(q,n)$.

Proof of Result 2.3. The probability that nobody volunteers will be larger for a group of size $n+1$ than for a group of size n if $q(\rho,n+1)^{n+1}>q(\rho,n)^n$. This will be the case if $q(\rho,n+1)>q(\rho,n)^{n/(n+1)}$. The equilibrium conditions require that

$$H\left(q(\rho,n+1),n+1\right)=H\left(q(\rho,n),n\right)=\rho. \tag{11}$$

According to Lemma 4,

$$H\left(q(\rho,n)^{n/n+1},n+1\right)<H(q(\rho,n),n). \tag{12}$$

Since, according to Lemma 2, $H(q,n+1)$ is an increasing function of q, it follows from Equations 11 and 12 that $q(\rho,n+1)>q(\rho,n)^{n/(n+1)}$ and hence $q(\rho,n+1)^{n+1}>q(\rho,n)^n$. This means that the probability that nobody volunteers increases with the number of players.

Let the probability no one volunteers be $Q(n,\rho)$. Since $Q(n,\rho)=q(n,\rho)^n$, the equilibrium condition in Equation 7 can be written equivalently as

$$\rho=\frac{n\left(1-Q(n,\rho)^{\frac{1}{n}}\right)Q(n,\rho)^{\frac{n-1}{n}}}{1-Q(n,\rho)} \tag{13}$$

which implies that

$$\left(1-Q(n,\rho)\right)\rho=n\left(Q(n,\rho)^{1-\frac{1}{n}}-Q(n,\rho)\right) \tag{14}$$

Making a change of variables $t=1/n$, Equation 14 can be written as

$$\left(1-Q\left(\frac{1}{t},\rho\right)\right)\rho=\frac{1}{t}\left(Q\left(\frac{1}{t},\rho\right)^{1-t}-Q\left(\frac{1}{t},\rho\right)\right). \tag{15}$$

Taking limits as $t\to 0$ and recalling that $\bar{Q}(\rho)=\lim_{n\to\infty}Q(n,\rho)$, yields the equation:

[2] Since $H(q,n)$ is an increasing function of q and for all $\rho\in(0,1)$, and in equilibrium $H\left(q(\rho,n),n\right)=\rho$, it must be that $q(\rho,n)$ is an increasing function of ρ.

$$\left(1-\bar{Q}(\rho)\right)\rho = \lim_{t\to 0}\frac{\bar{Q}(\rho)^{1-t}-\bar{Q}(\rho)}{t} \tag{16}$$

Where we define the function $g(x) = Q^x$, we have

$$\begin{aligned}\lim_{t\to 0}\frac{\bar{Q}(\rho)^{1-t}-\bar{Q}(\rho)}{t} &= lim_{t\to 0}\frac{g(1-t)-g(1)}{t}\\ &= -lim_{t\to 0}\frac{g(1-t)-g(1)}{-t}\\ &= -g'(1)\\ &= -\bar{Q}(\rho)\ln\bar{Q}(\rho)\end{aligned} \tag{17}$$

From Equations 16 and 17 it follows that

$$\rho = \frac{-\bar{Q}(\rho)}{1-\bar{Q}(\bar{\rho})}\ln\bar{Q}(\rho) \tag{18}$$

The function $\bar{Q}(\rho)$ can not in general be expressed in terms of standard elementary functions. However, it can be inverted using Lambert W function.

Note that *Equation 18* can be rewritten as:

$$-\frac{\rho}{Q}e^{-\frac{\rho}{Q}} = -\rho e^{-\rho} \tag{19}$$

This is of the form: $ye^y = x$ which can be inverted using Lambert W function and has solution:

$$-\frac{\rho}{Q} = W\left(-\rho e^{-\rho}\right) \tag{20}$$

We note that $-\frac{1}{e} < -\rho e^{-\rho} < 0$ for $0 < \rho < 1$. W is double-valued in the negative domain. However, along the principal branch (denoted W_0) $W_0(-\rho e^{\rho}) = \rho$. This implies $Q = 1$, which is not consistent with Nash equilibrium. Thus, only the minor branch (denoted W_{-1}) is solution associated with the family of symmetric Nash equilibria.

This can then be rearranged to get:

$$\bar{Q} = -\frac{\rho}{W_{-1}\left(-\rho e^{-\rho}\right)} \tag{21}$$

□

Appendix

2.6. *Proof of Lemma 2*

Proof of Lemma 2. The function $H(q,n)$ is a product of continuous functions of q and hence continuous. Since for $0 \le q < 1$,

$$\frac{1-q^n}{1-q} = 1 + q + q^2 + \dots q^{n-1}, \tag{22}$$

it must be that

$$\begin{aligned} H(q,n) &= \frac{nq^{n-1}}{1+q+q^2+\dots q^{n-1}} \\ &= \frac{n}{q^{1-n}+q^{2-n}+\cdots+1} \end{aligned} \tag{23}$$

The denominator of this expression is seen to be a decreasing function of q and thus $H(q,n)$ is an increasing function of q for all positive integers n. This proves Claim (i) of the lemma. Claim (ii) is easily verified by examination of Equation 23.

To establish Claim (iii), note that Equation 23 implies that

$$\frac{n+1}{H(q,n+1)} = \frac{n}{H(q,n)} + q^{-n} \tag{24}$$

Equation 24 implies that

$$\begin{aligned} n\left(\frac{1}{H(q,n+1)} - \frac{1}{H(q,n)}\right) &= q^{-n} - \frac{1}{H(q,n+1)} \\ &= q^{-n} - \frac{q^{-n}+q^{1-n}+q^{2-n}+\cdots+1}{n} \\ &> 0 \end{aligned} \tag{25}$$

where the final inequality in Expression 25 follows from the fact that when $0 < q < 1$, it must be that $q^{-n} > q^{i-n}$ for all $i > 0$. The inequality in Expression 25 implies that $H(q,n+1) < H(q,n)$, which is the assertion in Claim (iii). □

2.7. *Proof of Lemma 3*

Proof. Since

$$1 - x^n = (1-x)(1+x+x^2+\cdots+x^{n-1}) \tag{26}$$

and

$$1 - x^{n+1} = (1-x)(1+x+x^2+\cdots+x^n), \tag{27}$$

it follows that inequality 10 holds if and only if

$$\frac{x+x^2+\cdots+x^n}{1+x+x^2+\cdots+x^n} < \frac{n}{n+1} \tag{28}$$

Inequality 28 is equivalent to

$$x + x^2 + \dots x^n < n. \tag{29}$$

Since $0 < x < 1$, Inequality 29 applies. It follows that Inequality 10 holds.

□

2.8. *Proof of Lemma 4*

Proof of Lemma 4. Recalling that

$$H(q,n) = n\left(\frac{1-q}{q}\right)\left(\frac{q^n}{1-q^n}\right),$$

it must be that

$$H(q^{\frac{n}{n+1}}, n+1) = (n+1)\left(\frac{1-q^{\frac{n}{n+1}}}{q^{\frac{n}{n+1}}}\right)\left(\frac{q^n}{1-q^n}\right) \tag{30}$$

Therefore $H\left(q^{n/n+1}, n+1\right) < H(q,n)$ if and only if

$$(n+1)\left(\frac{1-q^{\frac{n}{n+1}}}{q^{\frac{n}{n+1}}}\right) < n\left(\frac{1-q}{q}\right) \tag{31}$$

or equivalently,

$$\frac{q^{\frac{1}{n+1}}\left(1-q^{\frac{n}{n+1}}\right)}{1-q} < \frac{n}{n+1} \tag{32}$$

Define $x = q^{\frac{1}{n+1}}$. Since $0 < q < 1$, it follows that $0 < x < 1$. Then Expression 32 can be written as

$$\frac{x(1-x^n)}{1-x^{n+1}} < \frac{n}{n+1} \tag{33}$$

Lemma 3 implies that this inequality holds and hence Inequality 32 is true.

□

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